

Chapter 2 Barnard's Star: Parallax, Proper Motion, And Possible Planetary Perturbation

2.1 INTRODUCTION

Barnard's Star is a fascinating object, not only because of its nearness and high proper motion, but also because of the earlier suggestions that it had planetary companions. Until the dramatic successes of the radial velocity searches for extrasolar planets, any hint of other planets attracted much interest. Such was the case with Barnard's Star. Its planetary saga began in 1963 with the first in a series of Sproul Observatory studies by van de Kamp suggesting the presence of a small amplitude astrometric perturbation. Later studies indicated that some portion of the perturbation was instrumental in nature and arose from a discontinuity in the imaging properties of the Sproul telescope. However, subsequent accounting for instrumental systematic errors in the Sproul data still yielded an astrometric perturbation, which could be represented by two circular orbits (van de Kamp 1975, 1977a, 1982). The most recent Leander McCormick Observatory (McCormick) investigation of Barnard's Star using photographic plate material is presented herein. Table 2.1 and Table 2.2 list the basic physical characteristics of Barnard's Star from the literature and from this study, respectively.

TABLE 2.1
PROPERTIES OF BARNARD'S STAR FROM LITERATURE

Property	Value	Reference
Position RA (hh mm ss.sss)	17 57 48.498	1
(2000.0) Declination (dd mm ss.ss)	+04 41 35.40	
Other Designations	BD +4°3561, LHS 57, GJ 699, G 140-24, NLTT 45718, TYC 425-2502-1, HIP 87937	2
Spectral Type	M4 V	3
Apparent V_{JM} Magnitude	9.53 \pm 0.03	4
Apparent K_{CIT} Magnitude	4.5 \pm 0.1	5
Luminosity (L_{\odot})	0.0035 \pm 0.0002	6
Radial Velocity (km s^{-1})	-110.5 \pm 0.4	7
Metallicity [m/H]	-0.5 \pm 0.5	8
Radius (R_{\odot})	0.200 \pm 0.008	6
Effective Temperature (K)	3,100 \pm 100	6
Population	Old Disk-Halo	5
	Intermediate population II	8

NOTE.—CIT system with $K=2.2 \mu\text{m}$ defined by (9)

REFERENCES.—(1) Tycho; (2) SIMBAD; (3) Kirkpatrick *et al.* 1991; (4) Weis 1993; (5) Leggett 1992; (6) Dawson & De Robertis 2004; (7) Nidever *et al.* 2002; (8) Gizis 1997; (9) Elias *et al.* 1982

TABLE 2.2
PROPERTIES OF BARNARD'S STAR FROM THIS STUDY

Property	Value	Comments
Parallax (mas)	552 \pm 7	
Proper Motion ($'' \text{yr}^{-1}$)	10.354 \pm 0.006	
Position Angle (degrees)	355.85 \pm 0.06	of proper motion
Secular Acceleration (mas yr^{-2})	1.253 \pm 0.041	
Distance (parsecs)	1.81 \pm 0.02	
Transverse Velocity (km/s)	89 \pm 1	
Absolute V_{JM} Magnitude	13.240 \pm 0.037	m_V from (1)
Absolute K_{CIT} Magnitude	8.2 \pm 0.1	m_K from (2)
Mass (M_{\odot})	0.16 \pm 0.04	K-band M-L relationship (3)
Luminosity (L_{\odot})	0.0032 \pm 0.0007	(4) and radius below
Radius (R_{\odot})	0.196 \pm 0.004	Radius-mass relationships (5, 6)

REFERENCES.—(1) Weis 1993; (2) Leggett 1992; (3) Delfosse *et al.* 2000 ; (4) Dawson & De Robertis 2004; (5) Caillault & Patterson 1990; (6) Neece 1984

2.2 HISTORY

E. E. Barnard discovered the star that bears his name in 1916 because of its large proper motion (Barnard 1916). In 1963, van de Kamp detected a perturbation in the proper motion of Barnard's Star that he explained by the presence of a planetary companion. Since that time, at least fourteen studies have attempted to confirm the presence of any planets. Although several techniques have been employed, none have been successful but only a few have been sensitive to low-mass objects in long-period orbits.

2.2.1 Astrometric Studies

Based on observations made with the Sproul Observatory 61-centimeter refractor between 1916 and 1962, van de Kamp's analysis had a planet with about 1.6 times the mass of Jupiter (M_{J1}) in an orbit with a period of 24 years and eccentricity of 0.6 (van de Kamp 1963b). In 1969, he updated his results with another single-planet solution and a two-planet alternative solution to the astrometric wobbles detected that fit the data equally well (van de Kamp 1969a, 1969b). Hershey's (1973) study of AC +65° 6955 indicated that physical changes made to the Sproul telescope in 1949 produced noticeable instrumental effects in plate series taken with the telescope that could give the appearance of a perturbation. After some accounting for these instrumental effects, van de Kamp (1975, 1977a, 1982) continued to assert that Barnard's Star had two planets. In his final 1982 analysis, he calculated that the detected planets had 0.7 and 0.5 M_{J1} with 12- and 20-year orbits, respectively. Van de Kamp (1986) estimated the errors on these masses to be $\pm 0.1 M_{J1}$.

Gatewood analyzed 610 exposures on 241 plates taken on 169 nights at the Allegheny and Van Vleck Observatories between 1916 and 1971 (Gatewood 1972, Gatewood & Eichhorn 1973). After accounting for differences between plates taken with different telescopes, Gatewood's analysis of the deviations between calculated and measured positions, or residuals, did not appear to indicate a perturbation greater than 11 milliseconds of arc (mas), which corresponds to a $1.2 \pm 0.2 M_{\text{J}}$ planet. In comparison, van de Kamp (1969a) used 10,452 exposures on 3,036 plates taken on 766 nights with a single instrument in his first 1969 analysis. Later investigation by Gatewood (1995) using his Multichannel Astrometric Photometer (MAP) hinted that a slight astrometric perturbation might exist in the proper motion of Barnard's Star. However, he thought the MAP residuals were most likely random measurement errors. According to Gatewood (1995), any planets present would have to be less massive than Jupiter or have either short or long orbital periods. Van de Kamp (1982) described just such planets.

Harrington and Harrington (1987) report an astrometric study of Barnard's Star at the United States Naval Observatory (USNO). Based on 400 plates taken in Flagstaff between 1972 and 1986, they reported that the residuals obtained appeared to approximate an unperturbed proper motion more closely than the periodic motion predicted by van de Kamp (1982). However, the data were inconclusive. In particular, observations made in 1977 were especially suggestive of a significant change in motion (Harrington & Harrington 1987). These observations cover a period slightly longer than the orbital period of the inner planet but less than a single orbit of the outer planet van

de Kamp described. When even a full orbital period has been observed, the linear component of a companion-induced perturbation may still be mistakenly included in the proper motion terms (Black & Scargle 1982).

Using the Fine Guidance Sensors (FGS) of the Hubble Space Telescope (HST), Benedict *et al.* (1999) observed Barnard's Star over three years. Using a periodogram technique similar to that described in section 2.3.3, they found significant power associated at a 0.5-year period. However, that signal appears to be associated with the rolling of the telescope. After additional corrections for motions of the telescope, they found no evidence of any companion with a mass slightly less than that of Jupiter and a period between 150 and 1,000 days, or between 0.4 and 3 years. This mass limit approaches that necessary to detect the planets described van de Kamp (1982). However, their observations and the periods they consider cover a fraction of the orbital periods van de Kamp calculated. This observation period is too short to decouple orbital and proper motion terms adequately if long-period planets were present.

Although McCormick has a substantial collection of photographic material for Barnard's Star, no previous investigation analyzed the complete collection. As shown in Figure 2.1, the preliminary analyses failed to detect any planetary companions (Fredrick & Ianna 1985; Ianna 1995; Bartlett & Ianna 2001) although some earlier reports (van de Kamp 1963a, 1969a; Harrington & Harrington 1987) hinted of a perturbation detected in this material.

A Barnard's Star Perturbation Search Using McCormick Observatory Photographic Plate Material

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TABLE I. PARALLAX AND PROPER MOTION

Parameter	Value
McCormick Parallax	740.2 ± 1.7 microarcseconds
Hubble Space Telescope Parallax	745.4 ± 0.9 microarcseconds
Tab Parallax	545.6 ± 1.1 microarcseconds
Hipparcos Parallax	549.3 ± 1.6 microarcseconds
McCormick Proper Motion	10132.4 ± 0.2 milliarcseconds/year
Hubble Space Telescope Proper Motion	10170.0 ± 0.3 milliarcseconds/year
Hipparcos Proper Motion	10160.6 ± 2.1 milliarcseconds/year

ABSTRACT

A time-series analysis of photographic plates taken of Barnard's Star at the Leander McCormick Observatory since 1969 reveals no evidence of periodic perturbations indicative of a planetary companion. Table I lists the proper motion and parallax of Barnard's Star calculated from these observations. Table II lists the resulting annual residuals, which are illustrated in Figure 1. The x- and y-residuals correspond to residuals in right ascension and declination, respectively. The error bars indicate a 95 percent confidence level. Figure 2 shows the periodograms corresponding to these residuals. In neither case does the power at any frequency indicate a signal at a significance level of 50 percent or better. However, this work does not absolutely rule out the possibility that Barnard's Star may have planets.

TABLE II. RESIDUALS FOR ANNUAL NORMAL POINTS

Year	X Residuals (microns)	Y Residuals (microns)	Images
1969	-0.0 ± 0.0	-0.3 ± 3.9	11
1970	-0.0 ± 0.0	-0.3 ± 0.7	50
1971	-0.2 ± 0.7	0.0 ± 0.7	41
1977	0.2 ± 0.4	0.3 ± 0.3	31
1978	-0.2 ± 0.3	0.3 ± 0.4	54
1979	0.1 ± 0.4	-0.7 ± 0.5	42
1980	0.4 ± 0.3	-0.1 ± 0.3	39
1981	-0.3 ± 0.4	-0.3 ± 0.4	45
1982	0.2 ± 0.4	-0.3 ± 0.4	50
1983	0.1 ± 0.4	-0.4 ± 0.4	84
1984	0.1 ± 0.9	-0.3 ± 0.7	21
1985	0.4 ± 0.9	-1.8 ± 1.1	8
1986	0.4 ± 0.5	0.0 ± 0.5	53
1988	-0.0 ± 0.7	0.0 ± 0.4	37
1989	0.5 ± 0.4	-0.3 ± 0.4	40
1990	-0.4 ± 0.3	0.0 ± 0.3	102
1992	-1.8 ± 0.9	-0.2 ± 1.2	8
1993	-1.4 ± 0.7	0.0 ± 0.9	17
1996	0.0 ± 0.0	-0.3 ± 0.7	59

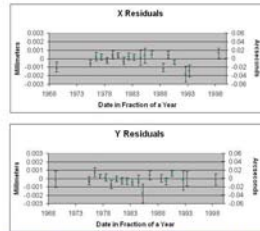


FIGURE 1. RESIDUALS FOR ANNUAL NORMAL POINTS

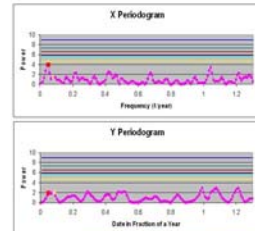


FIGURE 2. RESIDUAL PERIODOGRAMS

HISTORY

Barnard's Star is a particularly interesting object, not only because of its nearness and high proper motion, but also because of the early planet-detection claims. Table III lists several of the physical characteristics of this red dwarf. In 1963, Peter van de Kamp explained perturbations in its proper motion by the presence of a planetary companion. Based upon observations made at the Sproul Observatory between 1916 and 1962, Van de Kamp claimed the star had a planet with about 1.6 times the mass of Jupiter and an orbital period of 24 years. In 1969, he produced another single-planet solution and a two-planet solution to the astrometric wobble detected. After accounting for instrumentation effects that might have been partially responsible for his initial results, he continued to assert that Barnard's Star had two planets. Figures 3 and 4 illustrate the results of his 1982 analysis, in which he calculated that the detected planets had 0.7 and 0.5 Jupiter masses and 12- and 20-year orbits, respectively. However, the searches listed in Table IV have failed to confirm his results. Although the McCormick Observatory has a substantial collection of photographic material relating to Barnard's Star, these plates have only been partially analyzed in the past. The earlier analyses involving fewer observations also failed to detect any planetary companions.

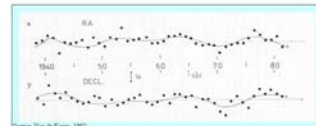


FIGURE 3. SPROUL RESIDUALS FOR ANNUAL NORMAL POINTS



FIGURE 4. SPROUL RESIDUALS AFTER REMOVING ORBITS

TABLE III. PHYSICAL CHARACTERISTICS

Parameter	Value
Distance	1.8 pc
Parallax	745.4 ± 0.9 microarcseconds
Proper Motion	10170.0 ± 0.3 milliarcseconds/year
Mass	$0.16 \pm 0.01 M_{\odot}$

TABLE IV. PARTIAL LIST OF BARNARD'S STAR STUDIES

Observer	Year of Publication	Method
Van de Kamp	1963, 1969, 1970, 1972	astrometric (Sproul plates)
Gerasimovic, Eshkolov	1977	astrometric (Astronomical and Van de Kamp plates)
Friedrich, Ianna	1987	astrometric (small set of McCormick plates)
Morley, Bessell	1989	radial velocity
Shkolnik et al.	1990	radial velocity
Barnard, Ianna	1990	astrometric (small set of McCormick plates)
Gerasimovic	1991	astrometric (Hubble Space Telescope plates)
Ianna	1991	astrometric (small set of McCormick plates)
Barnard et al.	1991	Hubble Space Telescope interferometry
Barnard, Ianna	2001	astrometric (complete set of McCormick plates)

TABLE V. MCCORMICK REFRACTOR CHARACTERISTICS

Parameter	Description
Overall System	Transmitted collection mounted by 10-in. Clark and Van de Kamp
Objective Lens	67 centimeters
Focal Ratio	f/14.5
Focal Length	969 centimeters
Angular Field	0.75 degrees
Plate Scale	20.75 arcseconds per millimeter
Barnard's Star Plates	101(4) plates with Vixen 112 film (100-100 sensitive load pair)
Mounting Equator	FES 100(100) Microdensitometer

TABLE VI. MCCORMICK OBSERVATIONS

Parameter	Value
Baseline	April 1969 through August 1996 (~27 years)
Images	924
Plates	204
Aligns	165
Reference Stars	12

TABLE VII. POWER SPECTRA PEAKS

Frequency (1/years)	Power	False Alarm Probability
X Residuals	1.1 ± 0.1	10%
Y Residuals	1.1	10%

METHOD AND RESULTS

The observers at McCormick Observatory obtained photographic plate images of Barnard's Star over many years with the 26.25-inch refractor. This analysis relies on these observations made since 1969 because the earlier material is too sparse and of such poor quality so as to be unusable for this study. Table V characterizes the McCormick refractor, which has been in operation since 1885. Table VI describes the observations upon which this analysis is based.

The McCormick flatbed microdensitometer measured the positions of Barnard's Star and 12 reference stars on each of the photographic plates. From these measurements, the McCormick Parallax Reduction Program calculated the proper motion and parallax of Barnard's Star, which are listed in Table I. The proper motion calculation included significant secular acceleration terms. The program output included the residuals associated with each observation. The residuals were combined into the annual normal points listed in Table II and illustrated in Figure 1. The periodograms of the x- and y-residuals were calculated separately using the Lomb method described in *Numerical Recipes in FORTRAN 77*. Frequencies up to four times the Nyquist frequency were searched to produce plots of power versus frequency. Figure 2 shows that the power at any frequency is indicative of a signal at a significance level of 50 percent or better. Table VII summarizes the characteristics of the highest peak in each power spectrum including the probability that the peak could be caused by noise. In both cases, the peaks are most likely false alarms. In addition, no signal is seen at the frequencies corresponding to 12 or 20 years, which are the periods associated with the planets reported by Van de Kamp in 1982.

DISCUSSION

This failure to replicate Van de Kamp's results, while not unexpected, does not completely eliminate the possibility that such planetary companions exist. The Sproul Observatory material available to Van de Kamp is unique. In 1982, his analysis used nearly 20,000 exposures on 4,580 plates taken on 1,200 nights between 1938 and 1981 at Sproul Observatory compared with this analysis of fewer exposures at McCormick Observatory. However, the McCormick observations were not subjected to significant physical changes, such as the modifications made to the Sproul refractor in 1949. In addition, the baseline of the McCormick observations represents only slightly more than twice the period of the more massive, but shorter period, planet described by Van de Kamp. Even with smoothing to annual normal points, the residuals to the proper motion and parallax solutions obtained in this study show the noise typical of photographic plates. Nonetheless, this study further reduces the possibility that planets as massive as Jupiter orbit Barnard's Star.

ACKNOWLEDGEMENTS

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Fig. 2.1 – Results of Preliminary Time-series Analysis of Photographic Plates from Leander McCormick Observatory (Bartlett & Ianna 2001).

2.2.2 Radial-Velocity Studies

Marcy and Benitz (1989) made twenty-five radial velocity observations of Barnard's Star over about four years as part of a larger search for brown dwarfs. After analyzing their short-interval measurements using a periodogram technique similar to that described herein, they reported no companions to Barnard's Star with projected masses (mass times the sine of an unknown inclination angle, $m \sin i$) greater than or equal to about $11 M_{\text{JL}}$. Their mass-detection sensitivity limit was many times greater than those calculated by van de Kamp (1982). Consequently, the California and Carnegie Planet Search continued radial velocity observations of Barnard's Star at the Lick and Keck Observatories. Their recent results appear to eliminate Jupiter-size planets with periods as long as eight years and also render a Jupiter-sized planet with an orbit of 11.5 years unlikely (C. McCarthy *et al.* 2006, in preparation).

Endl *et al.* (2002) used the European Southern Observatory Coudé Echelle spectrometer to take twenty-four radial-velocity measurements of Barnard's Star over nearly 4 years. Their observations are sensitive to planets within an astronomical unit of the central star, which is within the orbit of the inner planet described by van de Kamp (1982). They combined their estimated detectible mass limits with those of Benedict *et al.* (1999) in order to eliminate planets with projected masses ($m \sin i$) greater than $1.2 M_{\text{JL}}$. Van de Kamp's planets fall below this mass limit.

More radial velocity measurements of Barnard's Star, this time by Kürster *et al.* (2003), were made using the Ultraviolet-Visual Echelle Spectrograph on the 8.2-meter Kueyen Unit Telescope of the Very Large Telescope (VLT-UT2+UVES). Using a

periodogram technique, they searched the hydrogen-alpha (H_α) index and radial-velocity residuals for signals indicative of companions with periods between 2–885 days, or up to nearly 2.5 years. They found no evidence of planets with projected masses ($m \sin i$) greater than $0.12 M_{Jup}$. Within the habitable zone of Barnard's Star, which they associated with periods between 5.75 days and 21.5 days, they found no evidence of planets with projected masses as small as $0.02 M_{Jup}$. They acknowledge the possibility of missing massive planets with highly eccentric orbits. Although the masses excluded by this work would include those described by van de Kamp (1982), the orbital periods considered are far less than his.

2.2.3 Direct Imaging Studies

Skrutskie, Forrest, and Shure (1989) undertook the earliest direct imaging search. They observed stars in the solar neighborhood, including Barnard's Star, in the infrared in order to detect low-mass companions. They observed Barnard's Star on three nights in 1985. Because of saturating light from the primary star, companions within 2 seconds of arc ($2''$) of the primary were undetectable. At the distance of Barnard's Star, this separation equals a linear separation of about 4 astronomical units (AU). Van de Kamp (1982) calculated orbital radii of 2.7 and 3.8 AU for the planets; the inner planet would be undetectable but the outer planet would be near the edge of the saturation region. In addition, this survey was sensitive to substellar companions with an apparent K-band (effective wavelength of 2.2 micrometers, μm) magnitude brighter than 15.0. This magnitude limit corresponds to an object with an effective temperature of approximately 1,000 Kelvin (K) and more massive than $50 M_{Jup}$. Therefore, this search

would miss cooler planets less massive than Jupiter, such as those described by van de Kamp.

Oppenheimer *et al.* (2001) used coronagraphic images taken with Gunn r and z filters ($r = 668$ nanometers, nm, and $z = 912$ nm per Schneider, Gunn, & Hoessel 1983) and direct images taken with J ($1.25 \mu\text{m}$) and K filters to search for common proper motion companions to northern stars within 8 parsecs (pc) of the Sun. They took the optical images of Barnard's Star in 1992 and 1994 using the Palomar 60-inch telescope and infrared images in 1996 and 1997 using the Palomar 200-inch telescope. Although they found no companion to Barnard's Star, they detected six new stellar companions in other nearby systems. In z-band, their survey could conceivably detect brown dwarfs with ages of about 1 gigayear (Gyr) and with masses as small as $15 M_{21}$ separated from the primary star by distances between 20 and 225 AU. In J-band, their survey could possibly sense brown dwarfs with ages of about 1 Gyr and masses as small as $16 M_{21}$ separated from the primary star by distances between 5 and 120 AU. The separations and masses that Oppenheimer *et al.* (2001) would have revealed are both too large for their survey to detect van de Kamp's planets (1982).

Van Buren and his team (1998) undertook another infrared survey using an N-band filter ($10.2 \mu\text{m}$) at Palomar Observatory on two non-photometric nights. They were seeking brown dwarfs or giant planets orbiting within $2\text{--}10''$ of selected primary stars, which translates to a separation of $4\text{--}18$ AU for Barnard's Star. They did not detect any companion in this range with a temperature greater than 1,000 K, which would correspond to a planetary mass between 70 and $80 M_{21}$. Although the outer planet

described by van de Kamp (1982) is near the inner boundary of this search, he calculated a mass much smaller than the limit set by this survey.

In 1991 and again in 1998, Hinz *et al.* (2002) imaged Barnard's Star in J-band as part of a survey of northern M dwarfs within 8 pc of the Sun. They were seeking brown dwarfs sharing common proper motions with these stars. They detected no new companions brighter than approximately 16.5 magnitudes within about 4 minutes of arc of any the stars in their sample. Similar to the work of Oppenheimer *et al.* (2001), their survey was sensitive to 1-Gyr-old brown dwarfs with masses as low as $16 M_{24}$ (Hinz *et al.* 2002). The planets described by van de Kamp (1982) fall well below this limit.

Schroeder *et al.* (2000) used the HST wide field planetary camera 2 (WFPC-2) to image directly faint companions to the seventeen nearest star systems. They observed 23 stars within 13 pc of the Sun, including Barnard's Star on five occasions, using the F814W, F675W, and F1042M filters (F814W = 792.4 nm, F675W = 669.7 nm, F1042M \approx 1,019 nm per Holtzman *et al.* 1995). They found no new companions. The sensitivity of their search was insufficient to identify planets that Benedict *et al.* (1999) would have missed. Therefore, Schroeder *et al.* (2000) would not have detected the planets described by van de Kamp (1982) either.

2.2.4 Miscellaneous Studies

Although astrometric, radial-velocity, and direct imaging searches have been the most common approaches to detecting companions to Barnard's Star, other techniques have been used including radio observations and speckle interferometry.

Winglee, Dulk, and Bastian observed Barnard's Star at 0.33 Gigahertz (GHz) and 1.47 GHz with the Very Large Array (VLA) in 1986. They hoped to detect cyclotron maser radiation from Barnard's Star or its possible companions; several planets within our Solar System produce such radiation. However, they detected neither Barnard's Star nor any companion.

Using speckle interferometry, Henry and McCarthy (1990) sought brown dwarfs orbiting M dwarfs within 5 pc of the Sun. With respect to Barnard's Star, they looked for companions separated from it by a minimum of 2 AU, or about 1'', out to a maximum of 10 AU. The orbital radii calculated by van de Kamp (1982) fall within the inner half of this annulus. According to this study, any companion near the star would have to have an absolute K-band magnitude greater than 11.7 while any companion at the outer limit would have to have an absolute K-band magnitude greater than 12.2. However, their magnitude limits correspond to brown dwarfs with masses larger than $70 M_{24}$, which would miss the low-mass planets described by van de Kamp.

2.3 MEASUREMENT AND ANALYSIS

Observers at McCormick obtained Eastman Kodak 103a-G photographic plate images of Barnard's Star over many years with the 26.25-inch (67-centimeter) refractor and a Schott GG 495 filter giving a 495–600 nm passband (Schott 2006, Kodak 1989). The plate scale is $20.7484 \pm 0.0005''$ millimeter⁻¹ (Alden & van de Kamp 1927; Vyssotsky & van de Kamp 1937); the McCormick Parallax Reduction Program (sager; hereafter MPRP) uses the commonly adopted value of 20.75 mas millimeter⁻¹. On 107 nights between April 1969 and August 1998, observers obtained 919 good images on

293 plates. This analysis relies only on those observations made since 1969. The earliest material, including approximately sixty-three plates from 1916 to 1934 and eight plates from 1968, was excluded because it is too sparse and of inadequate quality to be usable for this study. In addition, different emulsions and filters were used from 1916 to 1934 (Mitchell *et al.* 1940). This exclusion eliminates all of the plates available to van de Kamp in 1963. The 1968 plates were among the first taken of Barnard's Star when it was reinstated in the parallax program at the request of P. A. Ianna (2006, private communication; L. Frederick 2006, private communication).

The McCormick Photometric Data Systems (PDS) 1010GM Microdensitometer, a granite-based and laser-encoded microdensitometer, measured the positions of Barnard's Star and twelve reference stars on each of the photographic plates. The microdensitometer scanned with a 20- μm square aperture and 10- μm step size to digitize a 71 by 71 pixel map centered on each reference star and Barnard's Star. Then, the relative image locations were obtained by fitting Gaussians to the sky subtracted marginal distributions. The plates were scanned in density mode in a boustrophedonic pattern. The repeatability of positions with this system is typically a few tenths of a micrometer and the accuracy of a single position with the somewhat noisy, coarse-grained emulsion is about $\pm 1.5 \mu\text{m}$. Guinan and Ianna (1983) provide further description of the reduction software.

2.3.1 Relative Motions

From these measurements, the MPRP calculated the relative parallax and proper motion of Barnard's Star to be $546.5 \pm 1.4 \text{ mas}$ and $10,361.25 \pm 0.20 \text{ mas yr}^{-1} (\text{yr}^{-1})$ in

355.9050 ± 0.0011 degrees ($^{\circ}$), respectively. The mean errors of unit weight were 1.3 and $1.5 \mu\text{m}$ in the x- and y-coordinates, respectively. Table 2.3 lists the individual components of each motion. The discrepancy of approximately 70 mas between the x- and y-components of the parallax, which correspond to right ascension and declination, appears to be due to the low signal in the y-direction. A very small-amplitude parallactic displacement is produced in the y-direction owing to the close proximity of Barnard's Star to the ecliptic. The calculation of relative parallax and proper motion used an iterative three plate-constant adjustment model, with scale, orientation, and origin terms. The program processed the reference star proper motions and plate constants twice to improve results. Several additional terms were tested using Tycho-2 (Høg *et al.* 2000, hereafter Tycho) V-band (510 nm) apparent magnitudes and B-V colors (B=420 nm). Individually, the coefficients of coma—magnitude times position—and color terms were not statistically significant as shown in Table 2.4. To be considered significant in a coordinate, a coefficient had to be greater than three times its associated error for the majority of exposures; the probability of such a comparatively large coefficient arising from noise alone is less than 1 percent (%) (Beers 1957). A magnitude term, however, was significant in x but not y. Including a magnitude term reduced the difference between the two parallactic components from 90 mas to 56 mas.

TABLE 2.3
COMPONENTS OF THE RELATIVE MOTION OF BARNARD'S STAR

Parameter	X		Y		Combined	
Parallax (mas)	547.9	± 1.4	474.6	± 9.9	546.5	± 1.4
Proper Motion (mas yr ⁻¹)	-739.9	± 0.2	10334.8	± 0.2	10361.25	± 0.20
Position Angle (degrees)		355.9050 ± 0.0011	
Secular Acceleration (mas yr ⁻²)	-0.025 ± 0.037		1.253 ± 0.041		1.253 ± 0.041	
Mean Error of Unit Weight (μ m)	1.3	...	1.5	
Mean Error of Unit Weight (mas)	27.1	...	32.1	

TABLE 2.4
ADDITIONAL PLATE MODEL TERMS CONSIDERED

Term	Significant in X?		Significant in Y?		Frames	Comments
	(Frames)	(%)	(Frames)	(%)	Considered	
V _T Magnitude	495	54	188	21	916	slight improvement in solution
Coma	393	43	74	8	916	
(B-V) _T Color	454	50	284	31	916	does not improve solution
V _T Mag & Acceleration	518	56	217	24	919	greatest overall improvement in solution
Coma & Acceleration	393	43	74	8	916	
(B-V) _T Color & Acceleration	454	50	281	34	916	solution quality similar to no acceleration

Because Barnard's Star has a large absolute parallax ($\pi = 552 \pm 7$ mas as calculated in section 2.3.2), proper motion ($\mu = 10.354 \pm 0.006'' \text{ yr}^{-1}$), and radial velocity ($V_r = -110.5 \pm 0.4$ kilometers second⁻¹ per Nidever *et al.* 2002), significant secular acceleration ($\dot{\mu}$) was expected (van de Kamp 1977b). The equation

$$\dot{\mu} = -2.05'' \times 10^{-6} V_r \mu \pi \quad (2.1)$$

predicts 1.29 ± 0.02 mas yr⁻². The geocentric motion of Barnard's Star may be described by

$$X = c_x + \mu_x t + \pi_x P_\alpha + q_x t^2 \quad (2.2)$$

$$Y = c_y + \mu_y t + \pi_y P_\delta + q_y t^2 \quad (2.3)$$

where c_x and c_y are geocentric positions at the mean time of the observations (1983.7740), t is time lapsed since then in solar years, P_α and P_δ are the parallactic factors in right ascension and declination, and q_x and q_y are half of the secular acceleration in each direction. We measured a secular acceleration of 1.253 ± 0.041 mas yr⁻²; Table 2.3 lists the individual components of this motion as well. The y-component of this secular acceleration was statistically significant. Table 2.5 compares this value with earlier studies. The current McCormick measurement of the secular acceleration of Barnard's Star is within the errors of the predicted value and that measured by Benedict *et al.* (1999) based on three years of HST observations. Although the inclusion of a secular acceleration term increased the difference between the x- and y-components of the relative parallax, it reduced the uncertainty in the y-component from 14 mas to 10

mas, decreased the number of residuals greater than or equal to $4\ \mu\text{m}$ from 72 to 33, and reduced the largest residual from $8.3\ \mu\text{m}$ to $4.8\ \mu\text{m}$.

TABLE 2.5
SECULAR ACCELERATION MEASUREMENTS OF BARNARD'S STAR

Measurement with Baseline	Secular Acceleration (mas yr ⁻¹)		Reference
Predicted	1.29	± 0.02	
McCormick, 1969–1998	1.25	± 0.04	
Hubble Space Telescope, 1993–1996	1.2	± 0.4	1
Sproul, 1916–1981	1.30	± 0.03	2
McCormick, 1916–1973	1.38	± 0.04	3

REFERENCES.—(1) Benedict *et al.* 1999; (2) van de Kamp 1986; (3) van de Kamp 1977b

Magnitude, coma, and color plate constants were also tested in conjunction with secular acceleration. Again, neither coma nor color terms made significant contributions to the solution as shown in Table 2.4. Using the magnitude term with secular acceleration produced a smaller discrepancy between two components of the parallax than using either no acceleration or acceleration alone; the difference was 68 mas rather than 90 mas or 100 mas. This combination of terms also produced a smaller error in y-component of the parallax than with no acceleration or with either acceleration or magnitude alone; the error was 9.7 mas rather than 14 mas, 10 mas, or 14 mas, respectively.

2.3.2 Reduction to Absolute

The relative parallax and proper motion of Barnard's Star are measured with respect to a reference frame of twelve stars, which are listed in Table 2.6 and shown in Figure 2.2. Several of these reference stars appear in other studies of Barnard's

TABLE 2.6
REFERENCE STARS

Reference Star	Tycho Identification	Coordinates (2000.0)		V_T (magnitude)	K_s (magnitude)	Other Designations
		Right Ascension (hh mm ss.sss)	Declination (dd mm ss.ss)			
1	TYC 425-262-1	17 57 51.948	+04 42 20.25	11.515 \pm 0.119	10.575 \pm 0.025	
2	TYC 425-1810-1	17 58 21.609	+04 36 09.90	11.100 \pm 0.087	5.963 \pm 0.017	
3	TYC 425-1467-1	17 58 56.021	+04 35 35.40	11.326 \pm 0.103	10.484 \pm 0.021	
4	TYC 425-1773-1	17 58 31.320	+04 30 53.67	11.838 \pm 0.181	9.917 \pm 0.021	
5	TYC 425-1397-1	17 58 36.252	+04 22 57.71	11.134 \pm 0.096	10.115 \pm 0.021	
6	TYC 425-802-1	17 58 24.420	+04 27 41.41	9.933 \pm 0.034	6.794 \pm 0.024	BD+4 3562, HIP 87991
7	TYC 425-22-1	17 57 19.878	+04 53 14.85	8.729 \pm 0.016	7.102 \pm 0.026	BD+4 3558, HIP 87901
8	TYC 425-187-1	17 56 43.618	+04 47 28.46	10.888 \pm 0.078	9.793 \pm 0.020	
9	TYC 425-1858-1	17 56 30.064	+04 37 37.66	10.785 \pm 0.071	8.983 \pm 0.022	
10	TYC 425-616-1	17 57 43.928	+04 26 23.60	8.831 \pm 0.016	8.648 \pm 0.023	BD+4 3560
11	TYC 425-214-1	17 57 34.938	+04 35 10.34	11.599 \pm 0.131	8.772 \pm 0.022	
12	TYC 425-1844-1	17 57 24.418	+04 36 09.29	10.522 \pm 0.049	8.676 \pm 0.022	BD+4 3559
Barnard's	TYC 425-2502-1	17 57 48.498	+04 41 35.40	9.776 \pm 0.028	4.524 \pm 0.020	BD+4 3561, HIP 87937

REFERENCES.—Positions and V_T magnitudes are from Tycho. K_s magnitudes are from 2MASS. Other designations are from SIMBAD.

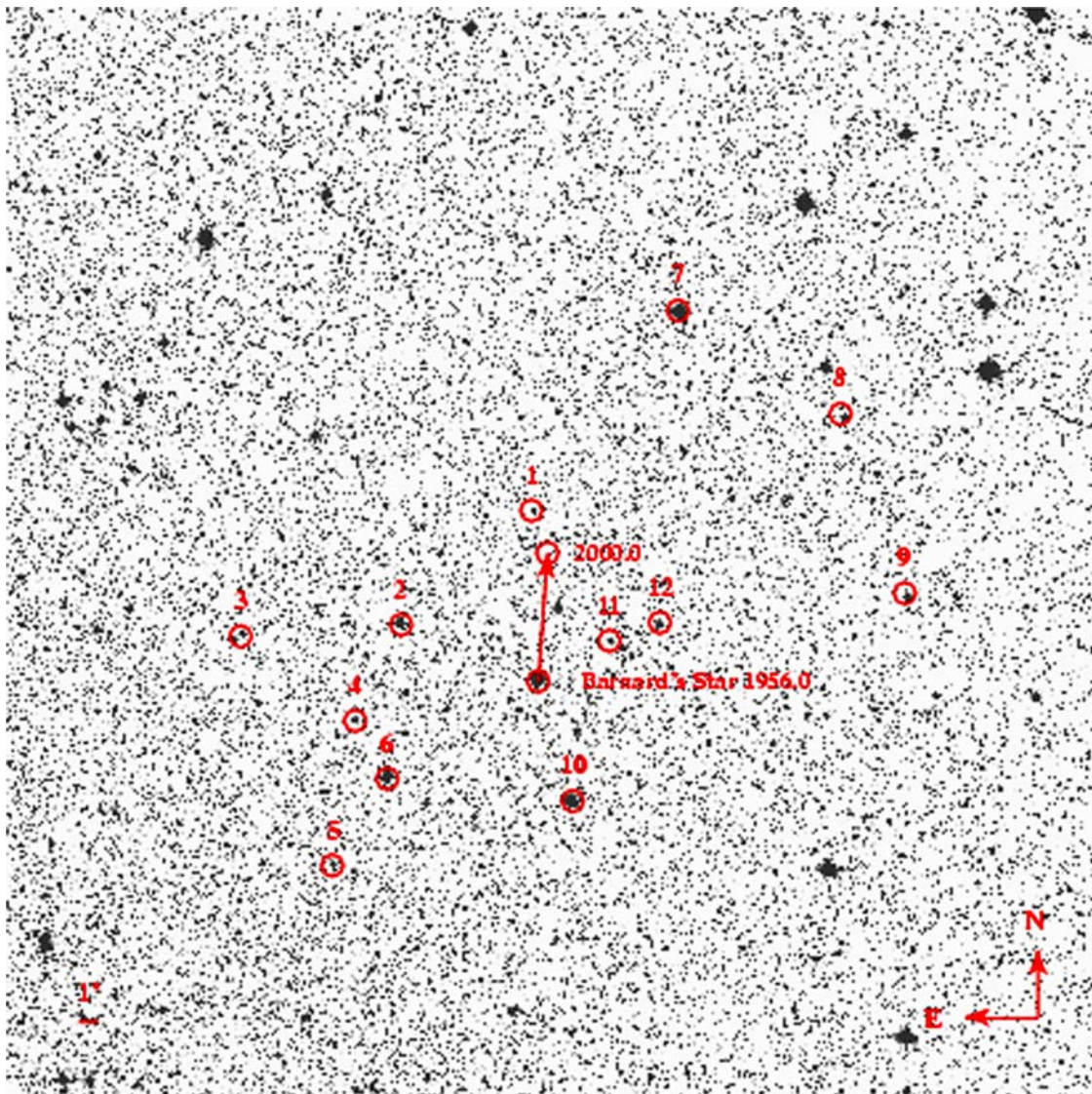


FIG. 2.2.— Barnard's Star (c. 1956.0) and Reference Frame. Image Source: Digitized Sky Survey via *SkyView* with annotations by R. Patterson, J. L. Bartlett, & J. L. Bartlett. Image Copyright © California Institute of Technology; digitized and compressed by Space Telescope Science Institute.

Star (van de Kamp 1935; Grossbacher, Mesrobian, & Upgren 1968; Gatewood & Eichhorn 1973; Castelaz *et al.* 1991; Benedict *et al.* 1999); Table 2.7 provides cross-references to the earlier studies.

Ideally, the reference stars should be sufficiently distant to show neither parallax nor proper motion themselves. In reality, these stars will have small parallaxes and proper motions over the period of observation. Therefore, a correction to account for the mean distance and motion of the reference frame is necessary to convert the relative parallax and proper motion to absolute ones. Because the MPRP corrects the measured positions for the relative proper motions of the reference stars, no additional correction is required for secular acceleration (van de Kamp 1977b).

Several methods exist for determining the necessary corrections to absolute parallax and proper motion ranging from statistical methods requiring little information about the individual reference stars to photometric and spectroscopic methods that are very specific to the reference stars used.

The Yerkes Galactic Model calculates mean and secular parallaxes based on the stellar luminosity function, the distribution of stars and dust, and solar and stellar kinematics. This model may be used to estimate corrections to convert relative parallaxes and proper motions to absolute ones using the galactic latitude and apparent visual magnitude of each reference star (van Altena 1974). To obtain comparable visual magnitudes, the Tycho V-band magnitudes were converted to the Johnson and Morgan (1951, 1953; hereafter Johnson-Morgan) system (VJM=550 nm) using the

TABLE 2.7
REFERENCE STAR CROSS-REFERENCES

Reference Star	Tycho Identification	van de Kamp ^a	Van Vleck Observatory ^b	Allegany Observatory ^c	Hubble Space Telescope ^d	References
1	TYC 425-262-1			13	36	1, 2
2	TYC 425-1810-1			16		1, 3
3	TYC 425-1467-1			20		1
4	TYC 425-1773-1					
5	TYC 425-1397-1					
6	TYC 425-802-1	4	2 nd π , BD +4°3562 ^e	17		4, 5, 1, 3
7	TYC 425-22-1			770		3
8	TYC 425-187-1			5		1
9	TYC 425-1858-1					
10	TYC 425-616-1	2	3, BD +4°3560	11		4, 5, 1, 3
11	TYC 425-214-1					
12	TYC 425-1844-1	1	2, BD +4°3559	10		4, 5, 1, 3

NOTES.— ^aReference star numbers used by van de Kamp in his various articles

^bVan Vleck Observatory reference star number and *Bonner Durchmusterung* (BD) identification

^cAllegany Observatory numbers

^dReference star identification number used by Benedict *et al.* 1999

^eVan Vleck Observatory used the same field to measure the parallax of Barnard's Star (BD +4°3561) and BD +4°3562.

REFERENCES.—(1) Gatewood & Eichhorn 1973; (2) Benedict *et al.* 1999; (3) Castelaz *et al.* 1991; (4) van de Kamp 1935; (5) Grossbacher, Mesrobian, & Upgren 1968

transformations in *The Hipparcos and Tycho Catalogues* (ESA 1997, hereafter Hipparcos). However, reference star 2 (TYC 425-1810-1) appears to be an M star (Castelaz *et al.* 1991) for which these transformations are not recommended. The V_{JM} magnitudes listed in Table 2.8 were assumed to correspond to van Altena's (1974) visual magnitude.

According to the Yerkes Galactic Model, 4.0 ± 1.6 mas should be added to the relative parallax and $1.7 \pm 6.3'' \text{ yr}^{-1}$ to the relative proper motion of Barnard's Star. *The General Catalogue of Trigonometric Stellar Parallaxes* (van Altena, Lee, & Hoffleit 1995, hereafter YPC) uses a more detailed galactic model (van Altena *et al.* 1988). Benedict *et al.* (1999) estimate their correction to absolute from Table 2 of this catalog. According to this table, a star at a galactic latitude of approximately 14.0° and with an average reference frame V magnitude of about 10.6 would require a correction of approximately 2.6 ± 0.2 mas added to the relative parallax (YPC; Benedict *et al.* 1999).

With Méndez, van Altena (1998) updated his galactic models once more. Méndez and van Altena offer to calculate the appropriate corrections upon request. In this instance, van Altena (2005, private communication) declined with the recommendation that calculating photometric distances for the reference stars and comparing their proper motions with those in the Hipparcos or Tycho catalogs would provide more reliable corrections.

Distances to stars may also be estimated assuming the apparent motion of their class is actually due to the motion of the Sun. Such secular parallaxes provide

TABLE 2.8
BRIGHTNESS OF REFERENCE STARS

Reference Star	Tycho Identification	B_{JM} (magnitude)	V_{JM} (magnitude)	K_{BB} (magnitude)	Comments
1	TYC 425-262-1	11.61 ± 0.19	11.50 ± 0.13	10.614 ± 0.026	
2	TYC 425-1810-1	12.71 ± 0.36	10.89 ± 0.15	6.001 ± 0.019	M3I star (ref 1), poor B and V transformations
3	TYC 425-1467-1	11.70 ± 0.17	11.28 ± 0.11	10.523 ± 0.022	
4	TYC 425-1773-1	12.38 ± 0.32	11.77 ± 0.20	9.956 ± 0.022	
5	TYC 425-1397-1	11.71 ± 0.16	11.07 ± 0.11	10.154 ± 0.022	
6	TYC 425-802-1	11.120 ± 0.080	9.792 ± 0.043	6.832 ± 0.025	
7	TYC 425-22-1	9.264 ± 0.028	8.666 ± 0.018	7.141 ± 0.027	
8	TYC 425-187-1	11.19 ± 0.13	10.852 ± 0.084	9.832 ± 0.021	
9	TYC 425-1858-1	11.42 ± 0.13	10.710 ± 0.079	9.021 ± 0.023	
10	TYC 425-616-1	8.845 ± 0.028	8.829 ± 0.017	8.687 ± 0.024	
11	TYC 425-214-1	12.57 ± 0.29	11.48 ± 0.16	8.810 ± 0.023	
12	TYC 425-1844-1	11.036 ± 0.087	10.461 ± 0.054	8.715 ± 0.023	
Barnard's	TYC 425-2502-1	11.178 ± 0.085	9.589 ± 0.039	4.562 ± 0.022	M4V star (ref 2), poor B and V transformations

REFERENCES.— B_{JM} and V_{JM} magnitudes from Tycho using Hipparcos transformations. K_{BB} magnitudes from 2MASS using Carpenter (2006) transformations. (1) Castelaz *et al.* 1991; (2) Kirkpatrick *et al.* 1991

another statistical approach to converting a relative parallax to an absolute one. Because the apparent solar motion varies with spectral type (Vyssotsky & Williams 1948b), knowledge of the spectral types of the reference stars is needed to convert the appropriate secular parallax to a mean parallax.

Because only five reference stars have published spectral types, spectral types were estimated from broadband photometry. Tycho B- and V-band apparent magnitudes were converted to the standard Johnson-Morgan system using the Hipparcos transformations. K_s -band ($2.159\ \mu\text{m}$) magnitudes from the Two Micron All Sky Survey (Skrutskie *et al.* 2006, hereafter 2MASS) were converted to the Bessell and Brett system of 1988 ($K_{BB}=2.19\ \mu\text{m}$) using the transformations from the *Explanatory Supplement to the 2MASS All Sky Data Release* (Carpenter 2006). Table 2.8 lists these transformed magnitudes. Then, the resulting B-V and V-K colors were de-reddened using dust maps derived from Diffuse Infrared Background Experiment (DIRBE) and Infrared Astronomy Satellite (IRAS) data with the Cerro Tololo Inter-American Observatory (CTIO) and United Kingdom Infrared Telescope (UKIRT) systems ($V_{CTIO}=551.9\ \text{nm}$ and $K_{UKIRT}=2.2152\ \mu\text{m}$) representing the standard optical and infrared bands (Schlegel, Finkbeiner, & Davis 1998).

The $(B-V)_0$ colors were compared to those provided by Drilling and Landolt in *Allen's Astrophysical Quantities* (2000, Table 15.7) to estimate spectral type and absolute V-band magnitude ($V_{DL}=550\ \text{nm}$) on a homogenized system closely related to the Johnson-Morgan system (Cousins 1976; Landolt 1992; Bessell 1979, 1976; Menzies *et al.* 1991). The $(V-K)_0$ colors were compared to those provided by Tokunaga in

Allen's Astrophysical Quantities (2000, Tables 7.6 through 7.8) to estimate spectral type and temperature. The resulting temperatures were used to calculate absolute V-band magnitudes (Drilling & Landolt 2000). Although the various transformations to V_{JM} magnitudes should result in consistent results, the definition of the Johnson-Morgan system itself evolved as it was applied to fainter and redder stars. Subsequent standards in that system by different observers show systematic differences (Menzies *et al.* 1991; Cousins 1984). The definition of K_{BB} appears less confused in the literature, but additional inconsistencies in those transformations may be revealed in time.

The assumption that all of the reference stars are main sequence stars produced reasonable distances, except for reference star 2 (TYC 425-1810-1) that appears to be a supergiant (Castelaz *et al.* 1991). The spectral types estimated from these colors were adopted for the reference stars without published spectral types. These estimated spectral types are probably accurate to within one spectral class. For those with published spectral types, the types determined by narrow-band photometry at Allegany Observatory (Castelaz *et al.* 1991), which are accurate to within 2.5 spectral subclasses, or determined by spectroscopy at the WIYN⁶ Observatory (Benedict *et al.* 1999) were adopted. Table 2.9 lists the broadband colors used, the initial estimated spectral types, and the final adopted spectral types.

Vyssotsky and Williams (1948a) provide two tables of secular parallaxes. In the first, they calculated secular parallaxes for stars grouped by photovisual magnitude

⁶WIYN stands for University of Wisconsin-Madison, Indiana University, Yale University, and National Optical Astronomy Observatories (NOAO).

TABLE 2.9
SPECTRAL TYPES FOR BARNARD'S STAR FIELD

Reference Star	Tycho Identification	(B-V) ₀ (mag)	(V-K) ₀ (mag)	Initial	van de Kamp	Kuiper	Spectral Types Allegany	Michigan	WIYN	Adopted
1	TYC 425-262-1	-0.0852	0.3759	A1V					A9III	A9III
2	TYC 425-1810-1	1.6140	4.3516	M3V			M3I			M3I
3	TYC 425-1467-1	0.2069	0.2089	A5V						A5V
4	TYC 425-1773-1	0.4136	1.2945	F4V						F4V
5	TYC 425-1397-1	0.4456	0.3883	F1V:						F1V:
6	TYC 425-802-1	1.1342	2.4479	K4V	F2	K4	K5V			K5V
7	TYC 425-22-1	0.3969	0.9915	F4V			G0V	F7V		G0V
8	TYC 425-187-1	0.0947	0.3682	A4V						A4V
9	TYC 425-1858-1	0.4720	1.0611	F5V						F5V
10	TYC 425-616-1	-0.1606	-0.3234	B6V	B8		B8I	B8/9III		B8I
11	TYC 425-214-1	0.8977	2.1648	K2V						K2V
12	TYC 425-1844-1	0.3828	1.2386	F4V	G0		G0V			G0V
Barnard's	TYC 425-2502-1	1.3938	4.5104	M3V						M4V

REFERENCES.—Spectral types from van de Kamp 1963b; Bidelman 1985 (Kuiper); Castelaz *et al.* 1991 (Allegany); Houk & Swift 1999 (Michigan); Benedict *et al.* 1999 (WIYN); and Kirkpatrick *et al.* 1991 (adopted for Barnard's Star).

and galactic latitude. In the second, they further divided the stars by spectral class. The secular parallaxes were converted to mean parallaxes using the solar velocities measured by Vyssotsky and Williams (1948b). Equating the V_{JM} and photovisual magnitudes and using the first table, a correction of 3.47 ± 0.43 mas should be added to the relative parallax to correct it to an absolute one using just the average reference frame brightness of 10.6 ± 1.0 magnitudes and galactic latitude of $30.93 \pm 0.11^\circ$. Averaging the corrections for each reference star should provide a more accurate correction. In this case, the first table produces a correction of 2.76 ± 0.85 mas to the relative parallax. According to the second table, the correction should be 3.2 ± 2.5 mas. In the latter two cases, the error represents the dispersion in mean parallaxes assigned to each reference star.

Photometric distances, which are calculated for each reference star, should give a better correction to convert a relative parallax to an absolute one than general statistical methods. Absolute V-band magnitudes were estimated from $(B-V)_0$ and $(V-K)_0$ when spectral types were determined for the reference stars. These values were converted to photometric parallaxes using the distance modulus formula and an interstellar absorption term (Schlegel, Finkbeiner, & Davis 1998). According to these distances, a correction of 6.8 ± 9.6 mas should be added to the relative parallax to convert it to an absolute parallax. These broadband photometric distances can be improved by the inclusion of trigonometric parallaxes from Hipparcos, narrow-band photometric distances from Allegany Observatory (Castelaz *et al.* 1991), and spectroscopic distances from WIYN (Benedict *et al.* 1999) where available. Using these

improved distances, a correction of 5.6 ± 6.7 mas is obtained. Table 2.10 lists broadband colors, apparent and absolute V-band magnitudes, estimated interstellar extinction, and parallaxes calculated from broadband photometry or provided by Hipparcos, Allegany, or WIYN.

The MPRP calculates relative proper motions for each of the reference stars as well as the star under investigation. The differences between the relative proper motions of the reference stars and known absolute proper motions provide a means of correcting the relative proper motion of Barnard's Star. Although absolute proper motions from Hipparcos are available for only two reference stars, absolute proper motions from Tycho are available for all twelve. According to the two Hipparcos values listed in Table 2.11, corrections of -20.6 ± 2.7 and -19.6 ± 2.2 mas yr⁻¹ should be added to the relative proper motion in right ascension and declination, respectively. According to the absolute proper motions from Tycho listed in Table 2.12, corrections of -9 ± 11 and -8.0 ± 5.6 mas yr⁻¹ should be added to the relative proper motion in right ascension and declination, respectively. The poor agreement between these corrections arises from the discrepancies in absolute proper motions available from these sources. As shown in Table 2.11 and Table 2.12, Hipparcos and Tycho report very different proper motions for two stars they have in common, reference stars 6 and 7 (TYC 425-802-1 and TYC 425-22-1). Tycho proper motions were calculated from a combination of ground-based observations and Hipparcos observations.

TABLE 2.10
PHOTOMETRIC PARALLAXES FOR REFERENCE STARS AND BARNARD'S STAR

Reference Star	Tycho Identification	(B-V) ₀ (mag)	(V-K) ₀ (mag)	V _{JM} (mag)	M _V (mag)	A _V (mag)	Photo.	Parallaxes (mas)			Final
								Hipparcos	Allegany	WIYN	
1	TYC 425-262-1	-0.0852	0.3759	0.5794	0.964	0.5794	1.0			1.0	1.0
2	TYC 425-1810-1	1.6140	4.3516	0.6055	-5.6	0.6055	0.1		0		0.0
3	TYC 425-1467-1	0.2069	0.2089	0.6207	1.853	0.6207	1.7				1.7
4	TYC 425-1773-1	0.4136	1.2945	0.5900	3.738	0.5900	3.2				3.2
5	TYC 425-1397-1	0.4456	0.3883	0.5908	2.652	0.5908	2.7				2.7
6	TYC 425-802-1	1.1342	2.4479	0.5777	6.898	0.5777	34.4	24.3	35		24.3
7	TYC 425-22-1	0.3969	0.9915	0.6016	3.607	0.6016	12.8	6.4	15		6.4
8	TYC 425-187-1	0.0947	0.3682	0.7352	1.784	0.7352	2.2				2.2
9	TYC 425-1858-1	0.4720	1.0611	0.7075	3.627	0.7075	5.3				5.3
10	TYC 425-616-1	-0.1606	-0.3234	0.5253	-0.958	0.5253	1.4		2		2.0
11	TYC 425-214-1	0.8977	2.1648	0.5735	6.228	0.5735	11.6				11.6
12	TYC 425-1844-1	0.3828	1.2386	0.5728	3.659	0.5728	5.7		7		7.0
Barnard's	TYC 425-2502-1	1.3938	4.5104	0.5828	9.728	0.5828	139.4	549.0			...

REFERENCES.—Parallaxes are from Hipparcos; Castelaz *et al.* 1991 (Allegany); and, Benedict *et al.* 1999 (WIYN).

TABLE 2.11
COMPARISON OF RELATIVE PROPER MOTIONS WITH HIPPARCOS PROPER MOTIONS

Reference Star	Tycho Identification	Relative Proper Motion (mas yr ⁻¹)		Hipparcos Proper Motion (mas yr ⁻¹)		Correction (mas yr ⁻¹)	
		Right Ascension	Declination	Right Ascension	Declination	RA	Dec
1	TYC 425-262-1	0.6 ± 0.2	-2.7 ± 0.2				
2	TYC 425-1810-1	7.5 ± 0.2	8.3 ± 0.2				
3	TYC 425-1467-1	20.1 ± 0.2	10.4 ± 0.2				
4	TYC 425-1773-1	16.3 ± 0.2	-3.8 ± 0.2				
5	TYC 425-1397-1	10.1 ± 0.3	2.6 ± 0.3				
6	TYC 425-802-1	-39.5 ± 0.1	-31.3 ± 0.2	-63.49 ± 2.46	-58.51 ± 1.93	-23.99	-27.21
7	TYC 425-22-1	0.5 ± 0.1	-2.6 ± 0.1	-16.74 ± 1.23	-14.68 ± 1.01	-17.24	-12.08
8	TYC 425-187-1	-6.3 ± 0.2	-8.3 ± 0.2				
9	TYC 425-1858-1	10.2 ± 0.2	-8.7 ± 0.2				
10	TYC 425-616-1	21.1 ± 0.1	8.4 ± 0.1				
11	TYC 425-214-1	-21.5 ± 0.3	-19.1 ± 0.2				
12	TYC 425-1844-1	-4.5 ± 0.2	38.7 ± 0.1				
					10326.9		
Barnard's	TYC 425-2502-1	-739.9 ± 0.2	10334.8 ± 0.2	-797.84 ± 1.61	3 ± 1.29

REFERENCE.—Hipparcos

TABLE 2.12
COMPARISON OF RELATIVE PROPER MOTIONS WITH TYCHO PROPER MOTIONS

Reference Star	Tycho Identification	Relative Proper Motion (mas yr ⁻¹)		Tycho Proper Motion (mas yr ⁻¹)		Correction (mas yr ⁻¹)	
		Right Ascension	Declination	Right Ascension	Declination	RA	Dec
1	TYC 425-262-1	0.6 ± 0.2	-2.7 ± 0.2	-3.6 ± 1.9	-6.4 ± 1.9	-4.2	-3.7
2	TYC 425-1810-1	7.5 ± 0.2	8.3 ± 0.2	-2.4 ± 2.3	0.8 ± 2.3	-9.9	-7.5
3	TYC 425-1467-1	20.1 ± 0.2	10.4 ± 0.2	4.2 ± 2.0	-3.0 ± 2.0	-15.9	-13.4
4	TYC 425-1773-1	16.3 ± 0.2	-3.8 ± 0.2	9.3 ± 2.1	-12.6 ± 2.0	-7	-8.8
5	TYC 425-1397-1	10.1 ± 0.3	2.6 ± 0.3	-8.8 ± 2.0	-6.4 ± 2.0	-18.9	-9
6	TYC 425-802-1	-39.5 ± 0.1	-31.3 ± 0.2	-62.4 ± 1.6	-46.3 ± 1.6	-22.9	-15
7	TYC 425-22-1	0.5 ± 0.1	-2.6 ± 0.1	-17.3 ± 1.0	-14.5 ± 1.1	-17.8	-11.9
8	TYC 425-187-1	-6.3 ± 0.2	-8.3 ± 0.2	0.7 ± 1.6	-5.9 ± 1.6	7	2.4
9	TYC 425-1858-1	10.2 ± 0.2	-8.7 ± 0.2	19.6 ± 1.9	-9.8 ± 1.8	9.4	-1.1
10	TYC 425-616-1	21.1 ± 0.1	8.4 ± 0.1	-2.1 ± 2.5	-6.2 ± 2.3	-23.2	-14.6
11	TYC 425-214-1	-21.5 ± 0.3	-19.1 ± 0.2	-19.7 ± 2.1	-27.9 ± 2.1	1.8	-8.8
12	TYC 425-1844-1	-4.5 ± 0.2	38.7 ± 0.1	-7.6 ± 1.7	34.0 ± 1.6	-3.1	-4.7
Barnard's	TYC 425-2502-1	-739.9 ± 0.2	10334.8 ± 0.2	-798.8 ± 1.6	10277.3 ± 1.5

REFERENCE.—Tycho

The various corrections proposed to convert the relative parallax and proper motion of Barnard's Star are summarized in Table 2.13 and Table 2.14, respectively.

The improved photometric distances were adopted for the parallax correction and the proper motions from Tycho were adopted for the proper motion correction because these values reflect the best available information on the reference stars overall.

Applying these corrections, Barnard's Star has an absolute parallax of 552 ± 7 mas and an absolute proper motion of $10.354 \pm 0.006'' \text{ yr}^{-1}$ in $355.85 \pm 0.06^\circ$, which puts it at a distance of 1.81 ± 0.02 pc with a transverse velocity of 89 ± 1 kilometers second⁻¹.

Table 2.15 compares the McCormick relative and absolute parallaxes with those available in the literature while Table 2.16 provides similar comparisons for proper motion.

TABLE 2.13
CORRECTIONS OF RELATIVE PARALLAX TO ABSOLUTE PARALLAX FOR BARNARD'S STAR

Technique	Correction (mas)		References
Mean Parallaxes, Yerkes Galactic Model	4.0	± 1.6	1
Yale Parallax Catalog	2.6	± 0.2	2
Secular Parallax of Mean Reference Frame	$3.47 \pm$	0.43	3, 4
Secular Parallaxes of Reference Stars (galactic longitude, apparent magnitude)	$2.76 \pm$	0.85	3, 4
Secular Parallaxes of Reference Stars (galactic longitude, apparent magnitude, spectral type)	$3.2 \pm$	2.5	3, 4
Photometric Parallaxes	$6.8 \pm$	9.6	5
Improved Photometric Parallaxes	$5.6 \pm$	6.7	5, 6, 7, 8

REFERENCES.—(1) van Altena 1974; (2) YPC; (3) Vyssotsky & Williams 1948a; (4) Vyssotsky & Williams 1948b; (5) see text for details; (6) Benedict *et al.* 1999; (7) Castelaz *et al.* 1991; (8) Hipparcos

TABLE 2.14
CORRECTIONS FROM RELATIVE TO ABSOLUTE PROPER MOTION FOR BARNARD'S STAR

Technique	Correction (mas yr ⁻¹)			Reference
Average Residual Proper Motions, Yerkes Galactic Model	1650	±	630	1
Hipparcos Absolute Proper Motions, Right Ascension	-20.6	±	2.7	2
Hipparcos Absolute Proper Motions, Declination	-19.6	±	2.2	2
Tycho Absolute Proper Motions, Right Ascension	-9	±	11	3
Tycho Absolute Proper Motions, Declination	-8.0	±	5.6	3

REFERENCES.—(1) van Altena 1974; (2) Hipparcos; (3) Tycho

TABLE 2.15
PARALLAX OF BARNARD'S STAR

Study	Parallax (mas)			Reference
McCormick, relative	546.5	±	1.4	
Hubble Space Telescope, relative	544.2	±	0.2	1
US Naval Observatory, relative	544.8	±	1.5	2
Sproul Observatory, relative	543.6	±	0.9	3
Van Vleck Observatory, relative	559	±	14	4
Allegany Observatory, relative	553	±	3.8	5
McCormick, absolute	552	±	7	
Hubble Space Telescope, absolute	545.4	±	0.3	1
Hipparcos, absolute	549	±	2	6
Yale Parallax Catalog, absolute	546	±	1	7
US Naval Observatory, absolute	547	±	2	2
Sproul Observatory, absolute	548	±	1	3

References.—(1) Benedict *et al.* 1999; (2) Harrington *et al.* 1993; (3) van de Kamp 1981; (4) Grossenbacher, Mesrobian, & Upgren 1968; (5) Beardsley *et al.* 1973; (6) Hipparcos; (7) YPC

TABLE 2.16
PROPER MOTION OF BARNARD'S STAR

Study	Proper Motion (mas yr ⁻¹)	Position Angle (degrees)	Ref.
McCormick, relative	10,361.3 ± 0.2	355.905 ± 0.001	
Hubble, relative	10,370.0 ± 0.3	355.6 ± 0.1	1
Yale Parallax Catalog, relative	10,310 ...	356 ...	2
US Naval Observatory, relative	10,360.3 ± 0.3	355.8 ± 0.0	3
Sproul, relative	10,307.0 ± 0.1	355.5277 ± 0.0006	4
Allegany, relative	10,318.7 ± 0.3	356.044 ± 0.002	5
Van Vleck, relative	10,349 ± 7	355.98 ± 0.07	6
McCormick, absolute	10,354 ± 6	355.85 ± 0.06	
Tycho, absolute	10,308 ± 2	355.556 ± 0.009	7
Hipparcos, absolute	10,358 ± 1	355.582 ± 0.009	8

REFERENCES.—(1) Benedict *et al.* 1999; (2) YPC; (3) Harrington *et al.* 1993; (4) van de Kamp 1981; (5) Beardsley *et al.* 1973; (6) Grossenbacher, Mesrobian, & Upgren 1968; (7) Tycho; (8) Hipparcos

The relative parallax for Barnard's Star from McCormick falls towards the middle of the published relative parallaxes from other programs. It is slightly larger than that measured by the Benedict *et al.* (1999), but its lower limit is within the upper range of the relative parallax measured at the USNO (Harrington *et al.* 1993). The absolute parallax from this study is larger than any of the absolute parallaxes found in literature; however, its lower limit is within the upper range of all the published values.

The absolute parallax resulting from this study suffers from a large correction to absolute (5.6 mas), nearly three times greater than used by the USNO (Harrington *et al.* 1993) and nearly five times greater than used by Benedict *et al.* (1999), along with a large associated error (± 6.7 mas). Benedict *et al.* (1999) obtained their correction to absolute from the YPC, which was within 0.2 mas of spectrophotometric correction. Van Altena (2005, private communication), the primary author of the YPC, discouraged

such a statistical approach in favor of a more tailored approach, such as photometric or spectroscopic distance estimates for the actual reference stars used. The reference frame selected includes two stars within 100 pc, reference stars 6 (TYC 425-802-1) and 11 (TYC 425-214-1), without which the correction would be reduced by more than a half. Based on the parallaxes listed in Table 2.17, reference star 6 has a parallax of 25 ± 2 mas, which puts it as a distance of about 41 pc. Removing reference star 6 from the analysis degrades the parallax solution. The distance estimate for reference star 11, approximately 86 pc, used herein relies solely on broadband photometry. Recent spectra obtained for this star with the 40-inch telescope at Fan Mountain Observatory may increase this distance estimate, reduce the correction to absolute, and improve the absolute parallax obtained. Alternatively, the example of Benedict *et al.* (1999) could be followed and the correction estimated from the YPC be adopted, which would result in an absolute parallax of 549 ± 1 mas.

TABLE 2.17
PARALLAX OF REFERENCE STAR 6 (TYC 425-802-1)

Study	Parallax (mas)			Reference
McCormick, relative	16	\pm	1	byproduct of this analysis
Hipparcos, absolute	24.32	\pm	2.28	1
Yale Parallax Catalog, absolute	27.2	\pm	2.8	2
Allegany, absolute	22	\pm	3	3

NOTE.—Reference star 6 (TYC 425-802-1) is also known as HIP 87991, Plx 4102, Allegany 17, BD +4°3562, and Kuiper 84.

REFERENCES.—(1) Hipparcos; (2) YPC; (3) Gatewood 1973

The relative proper motion of Barnard's Star measured at McCormick is larger than all the other published measurements except that by Benedict *et al.* (1999). The relative proper motions calculated by McCormick, the USNO (Harrington *et al.* 1993),

and Benedict *et al.* (1999) do not overlap even considering all their formal errors. The position angles associated with the published proper motions all lie within 5° west of due north with the value measured herein towards the middle of the published values. In most published studies, the observers have been content to provide relative proper motions and position angles without attempting to correct these measurements for any motion in the reference frame. Hipparcos and Tycho provide absolute proper motions and position angles; their proper motions bracket the value measured at McCormick. The McCormick position angle indicates a slightly more northerly motion than the other two measurements. However, the McCormick relative proper motion was converted to an absolute one through comparison with Tycho proper motions.

2.3.3 Time-Series Analysis

The MPRP also calculates the residuals associated with each observation, which are shown in Figure 2.3. Lacking an obvious periodic signal, a time-series analysis of these residuals should reveal whether they result from random measurement error or from a periodic perturbation such as van de Kamp (1982) described. The frequency domain provides mathematical tools for identifying a signal with an unknown period from a collection of measurements. When observations occur at uneven intervals, as is the case with the photographic plates in this study and for many other astronomical observations, a least-squares spectrum reveals the relative spectral power that each evaluated frequency, or its associated period, contributes to the measurements

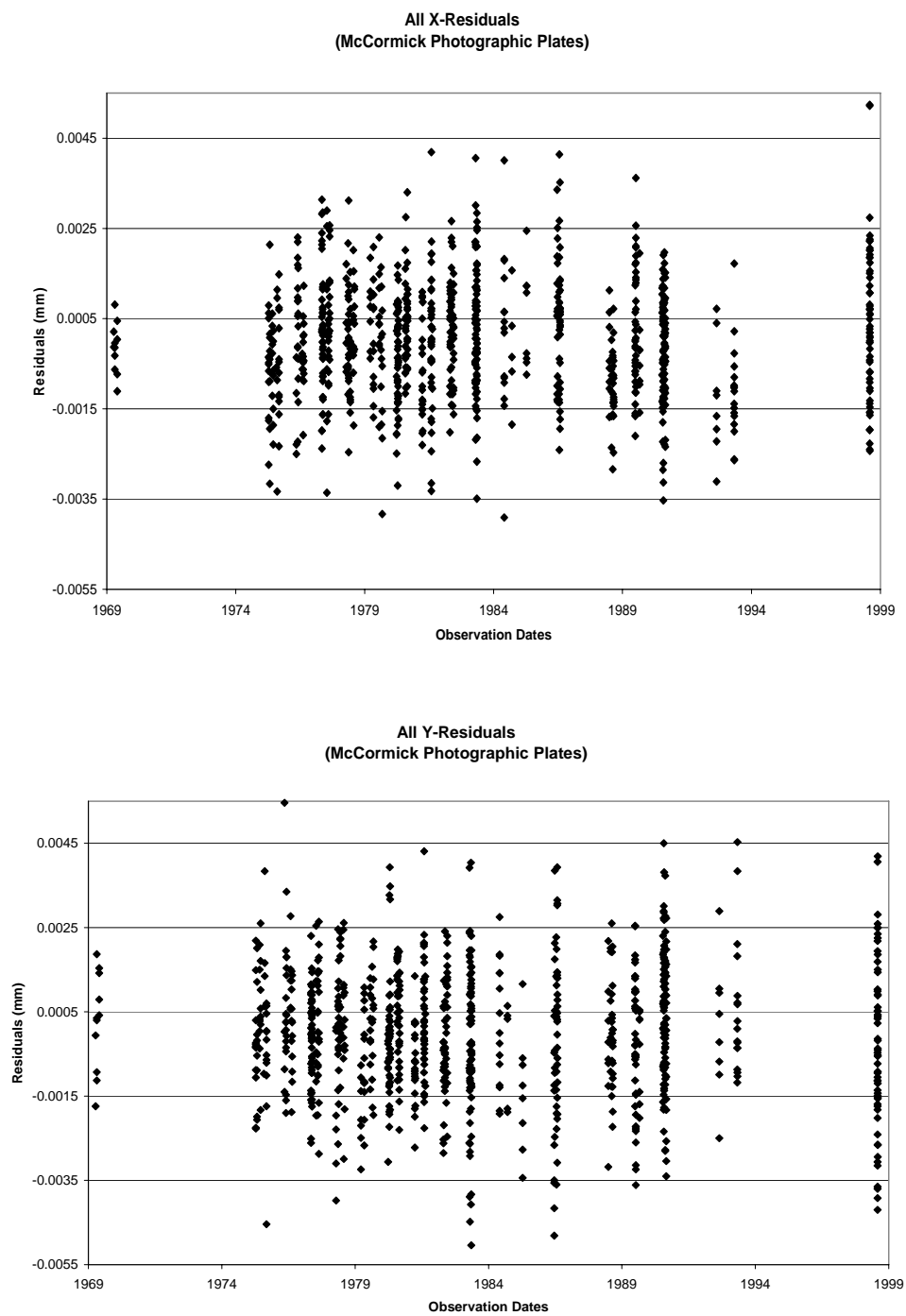


FIG. 2.3.—All X- and Y-Residuals for Barnard's Star. Top chart contains all x-residuals. Bottom chart contains all y-residuals.

(Barning 1963; Lomb 1976). This least-squares spectrum is calculated by fitting sine waves to the data so as to minimize the squares of the resulting residuals (Barning 1963). Alternatively, the classical periodogram, which works well for evenly sampled data, provides estimates of the spectral power of tested frequencies in irregularly sampled sets (Lomb 1976).

However, the classical periodogram may be modified in such a way that it becomes equivalent to the corresponding least-squares spectrum; this modified periodogram, known as the Lomb-Scargle normalized periodogram, reduces to the classical one when even time intervals are used (Scargle 1982). Press and Rybiki (1989) reduced the computational burden of calculating the Lomb-Scargle normalized periodogram by replacing single measurements with multiple, evenly spaced values that approximate the original quantity.

To test this approach, the Ephemeris2 program was altered to calculate the positions of planets van de Kamp (1982) described for the time of each observation using the modified Thiele-Innes constants he provided, which are listed in Table 2.18.

TABLE 2.18
ORBITAL PARAMETERS FOR VAN DE KAMP'S PLANETS

Parameter	Primary Solution		Single Planet Solution
	Inner Planet	Outer Planet	
(B) in milliseconds of arc	2.1 \pm 1.1	-4.0 \pm 1.1	2.1 \pm 1.1
(G) in milliseconds of arc	5.3 \pm 1.1	-0.9 \pm 1.1	6.4 \pm 1.1
(A) in milliseconds of arc	-0.8 \pm 1.3	-4.5 \pm 1.3	-2.1 \pm 1.3
(F) in milliseconds of arc	4.5 \pm 1.3	-3.4 \pm 1.3	5.5 \pm 1.3
Period in years	12.0 \pm 1.0	20 \pm 3	12.0 \pm 0.5
Eccentricity	0.0	0.0	0.0
Periastron Time	N/A	N/A	N/A

REFERENCE.—van de Kamp 1982

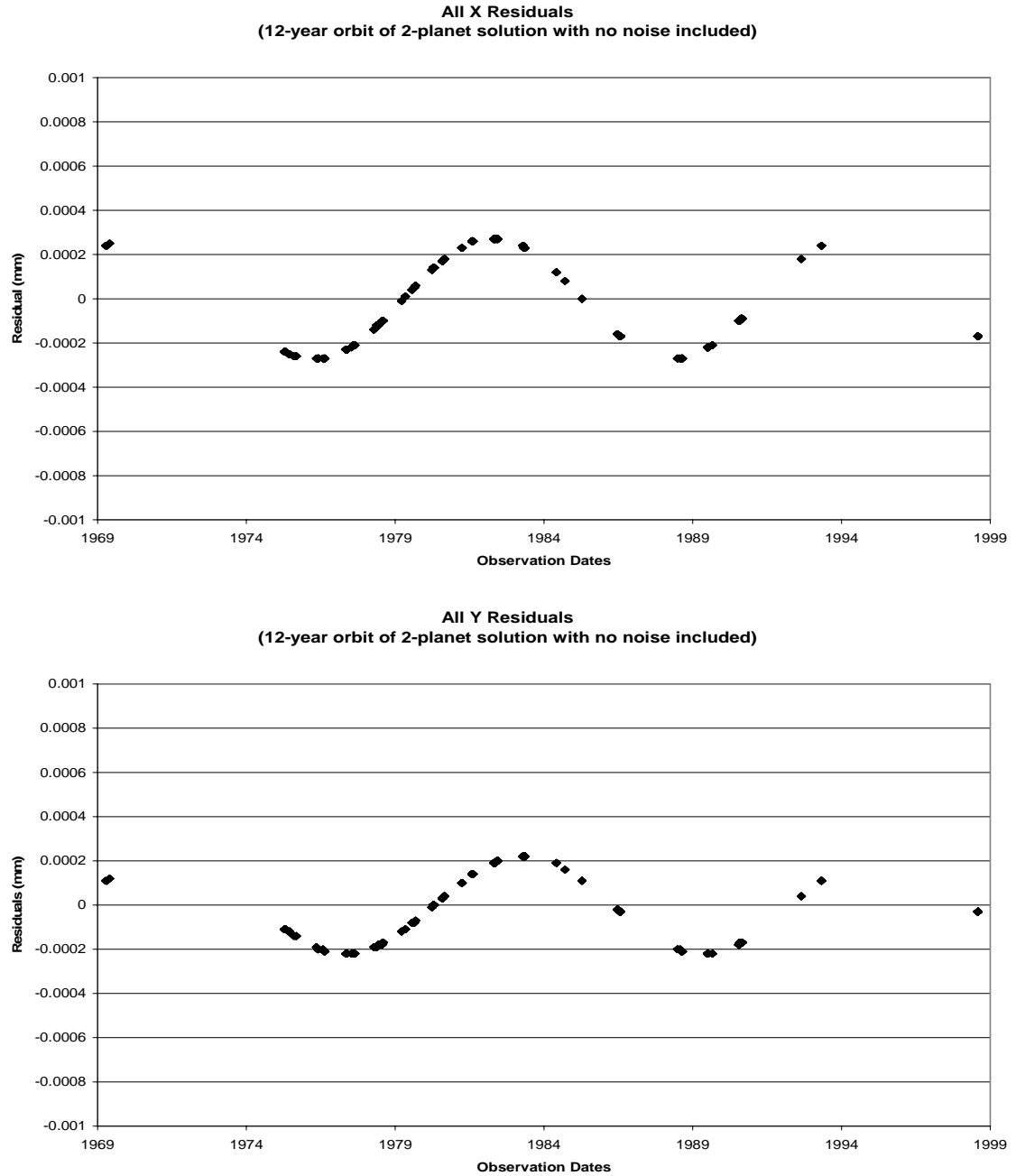


FIG. 2.4.— Theoretical Displacement of Barnard's Star due to Planet in 12-year Orbit Described by van de Kamp (1982). Top chart contains theoretical x-residuals resulting from the perturbation alone. bottom panel contains theoretical y-residuals.

Figure 2.4 shows the displacement of Barnard's Star due to the inner planet of the two-planet solution, ignoring any additional perturbation due to the outer planet and without the addition of any noise. Their periodic signals were recovered by calculating separate Lomb-Scargle periodograms for the x- and y-positions using the spreading technique of Press and Rybiki (1989; Press *et al.* 1996). The 12-year period is easily recovered from the corresponding periodograms in Figure 2.5.

When separate Lomb-Scargle periodograms were calculated for the x- and y-residuals produced by MPRP, the periodograms shown in Figure 2.6 resulted. These periodograms are very noisy and show the spurious peaks expected when a large number of frequencies are searched (Scargle 1982).

Scargle (1982) discusses two ways of improving detection efficiency: obtaining additional observations and averaging the data. The former method was not an option because the photographic plates used in this study are no longer manufactured. Therefore, we resorted to the latter by combining the residuals into nightly and annual normal points and re-calculating periodograms for the averaged data. Figure 2.7 and Figure 2.8 show the nightly normal points and the corresponding periodograms. Table 2.19 lists the annual normal points, which are illustrated in Figure 2.9. The Nyquist frequency is the highest frequency that can be recovered at a given even sampling rate. Averaging the data reduces the effective Nyquist frequency, which may drop below the signal frequency making detection more difficult (Scargle 1982). Fortunately, as shown in Table 2.20, the frequency of either a 12- or a 20-year period perturbation

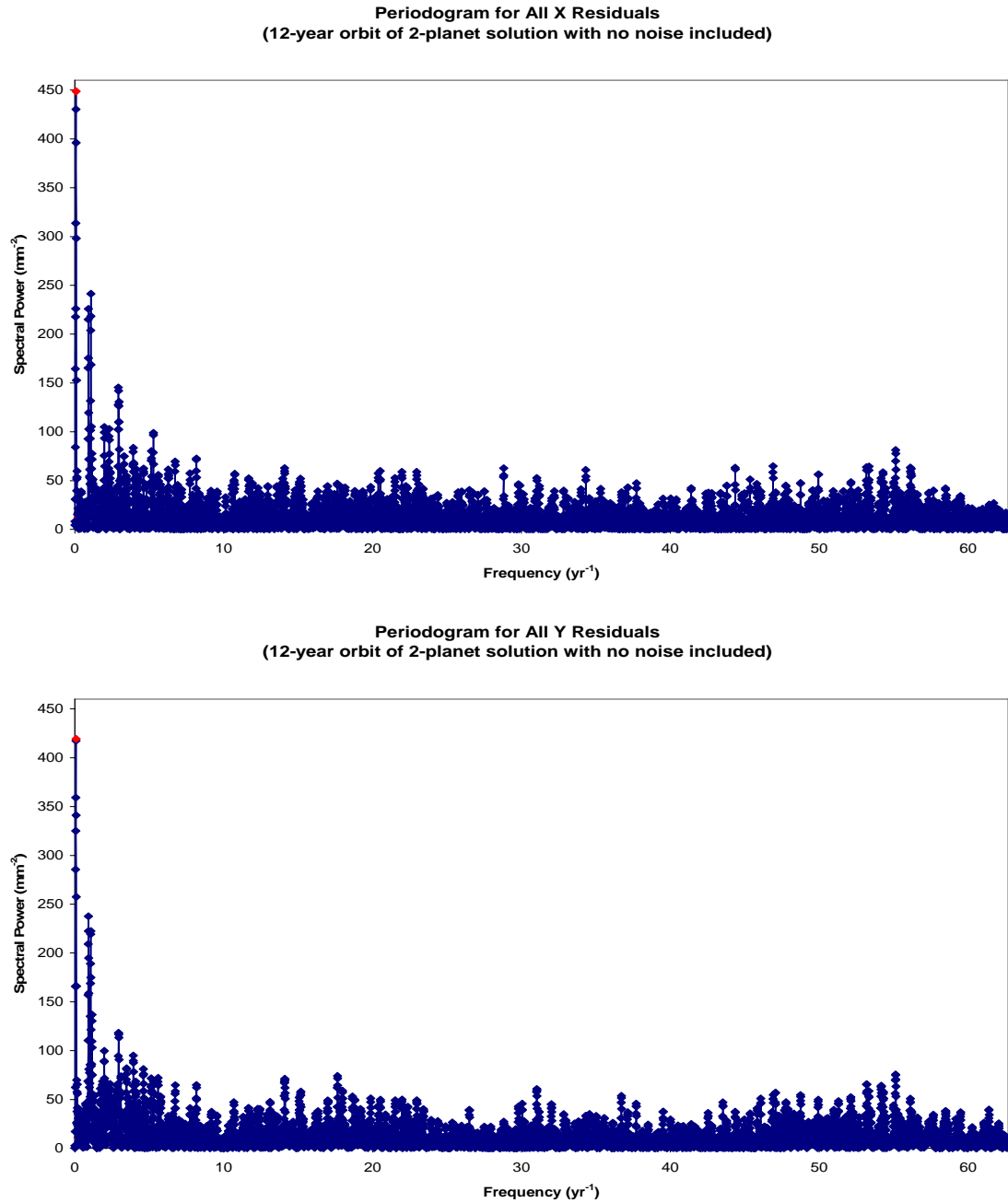


FIG. 2.5.—Periodograms of Theoretical Displacement of Barnard's Star due to Planet in 12-year Orbit Described by van de Kamp (1982). Top chart is a periodogram calculated for theoretical x-residuals. Bottom chart is a periodogram calculated for theoretical y-residuals. The maxima in both periodograms at 0.0853 yr^{-1} (11.7 years) tower so far above the 99% confidence level at 12.8 mm^2 that the confidence levels cannot be distinguished on this scale.

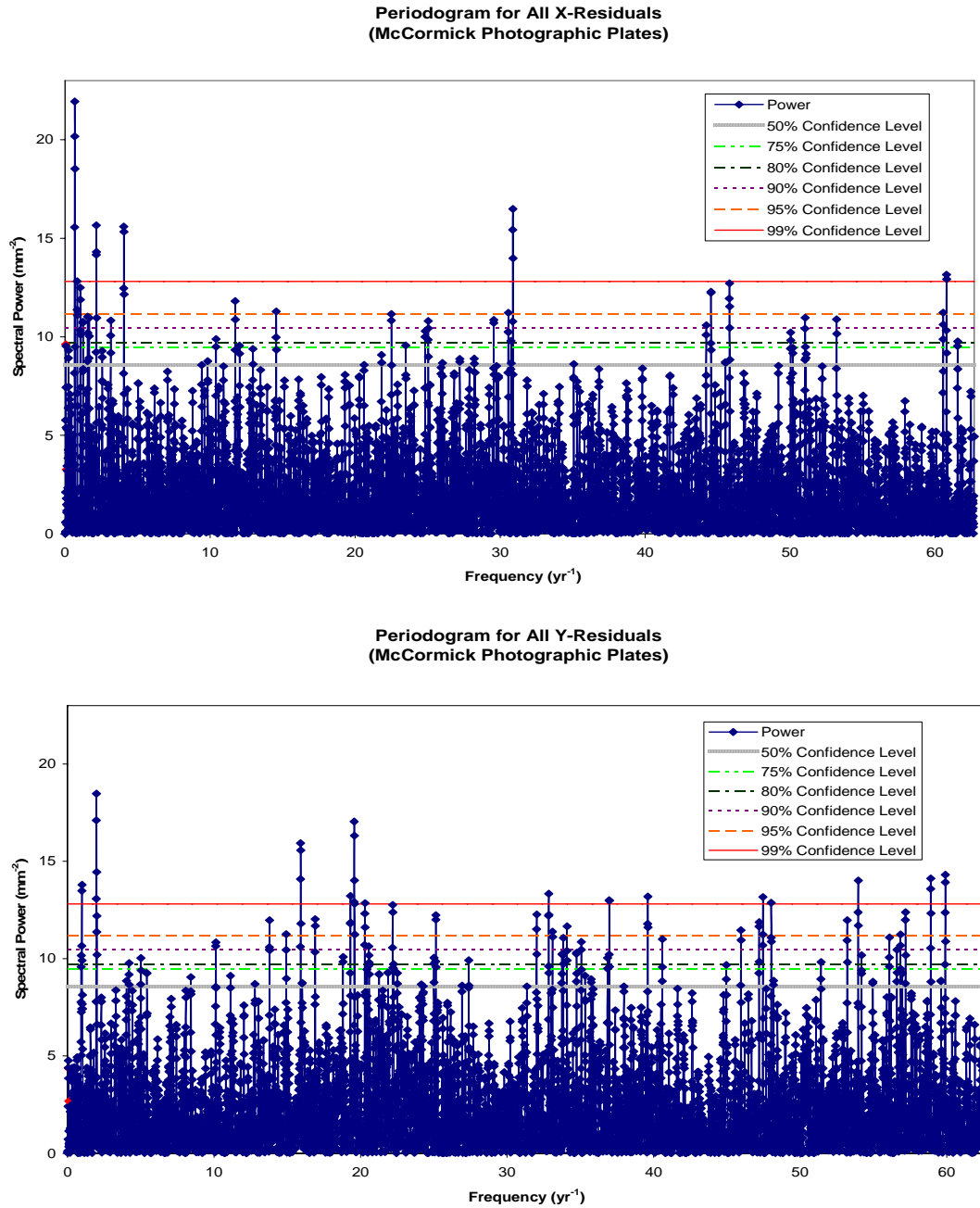


FIG. 2.6.— Periodograms for All X- and Y-Residuals for Barnard's Star. Top chart is a periodogram calculated for all x-residuals. Bottom chart is a periodogram calculated for all y-residuals. Horizontal lines in the periodograms indicate likelihood that spectral peaks exceeding that power are real and not caused by noise only. Solid line is 99% confidence, dashed line 95% confidence, dotted line 90% confidence, dash dotted line is 80% confidence, dash double-dotted line is 75% confidence, and hashed line is 50% confidence.

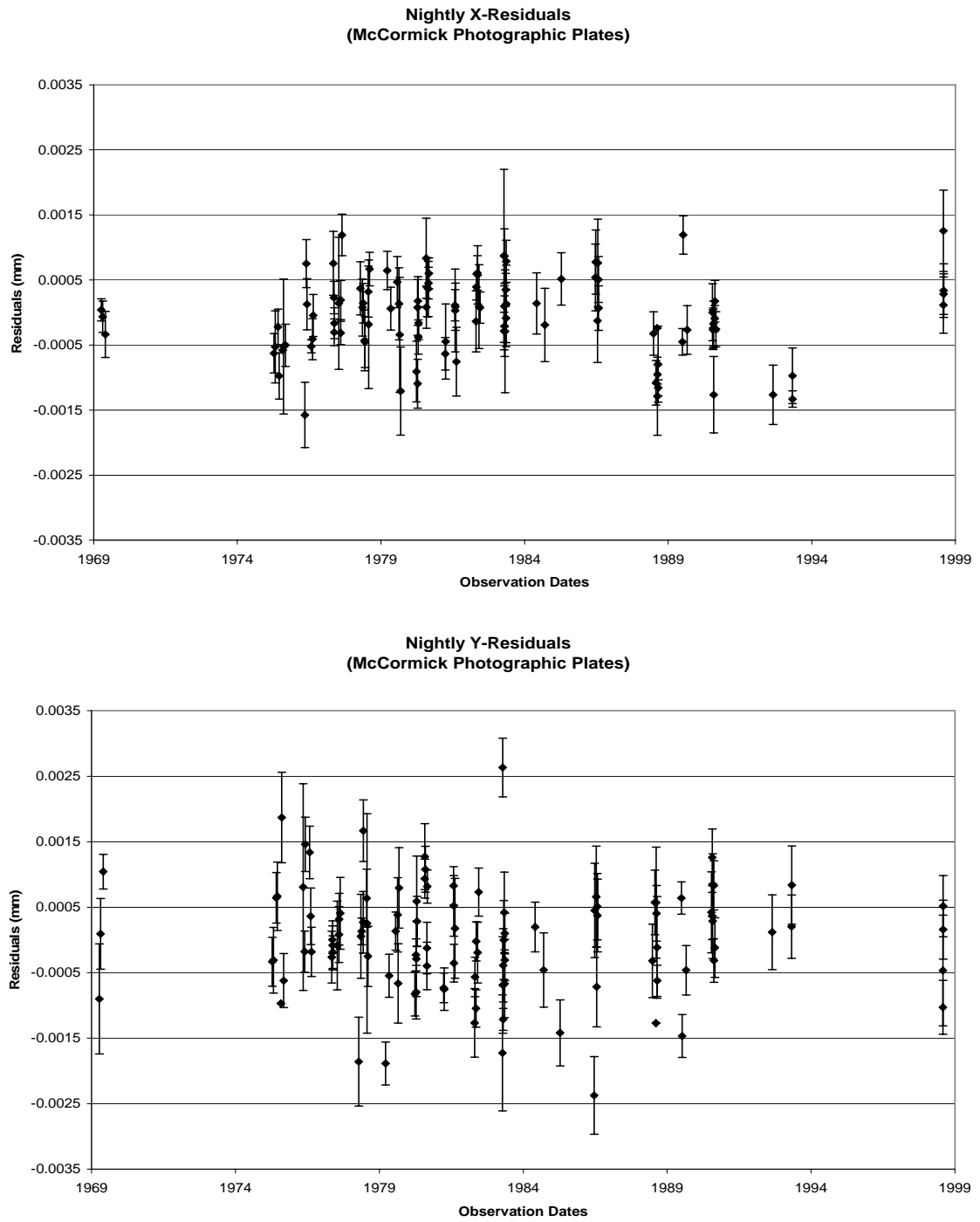


Fig. 2.7.— Nightly X- and Y-Residuals for Barnard's Star. Top chart contains x-residuals averaged by night. Bottom chart contains y-residuals averaged by night.

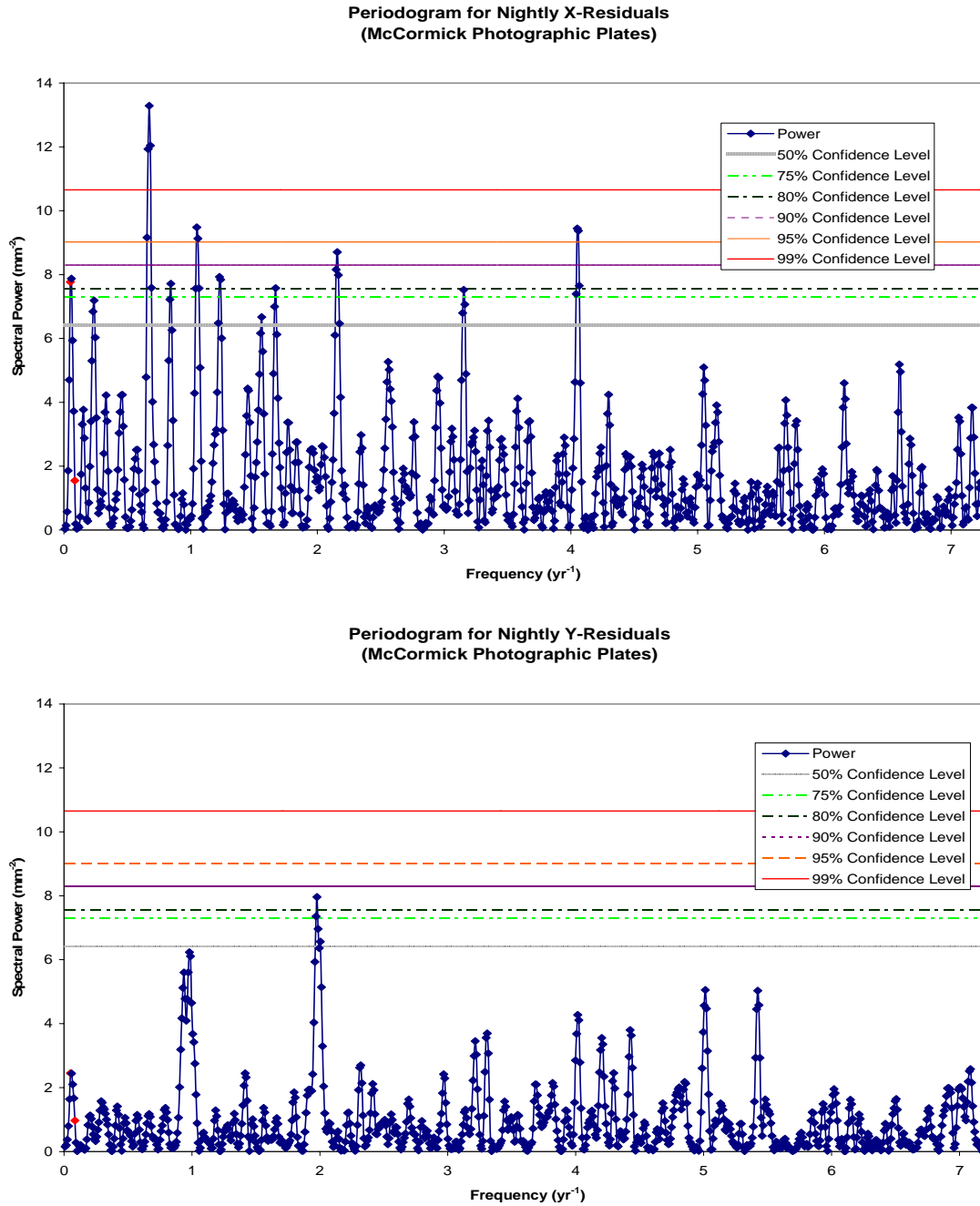


Fig. 2.8.— Top chart is a periodogram calculated for nightly x-residuals. Bottom chart is a periodogram calculated for nightly y-residuals. Horizontal lines in the periodograms indicate likelihood that spectral peaks exceeding that power are real and not caused by noise only. Solid line is 99% confidence, dashed line 95% confidence, dotted line 90% confidence, dash dotted line is 80% confidence, dash double-dotted line is 75% confidence, and hashed line is 50% confidence.

TABLE 2.19
ANNUAL NORMAL POINTS FOR BARNARD'S STAR

Observations (Year Fraction)	X Residuals (millimeters)	Y Residuals (millimeters)	Images
1969.339 \pm 0.017	-0.00015 \pm 0.00017	0.00026 \pm 0.00035	11
1975.455 \pm 0.022	-0.00057 \pm 0.00016	0.00002 \pm 0.00021	50
1976.494 \pm 0.017	-0.00005 \pm 0.00018	0.00047 \pm 0.00023	41
1977.458 \pm 0.013	0.00028 \pm 0.00015	0.00000 \pm 0.00013	81
1978.431 \pm 0.013	0.00009 \pm 0.00015	0.00009 \pm 0.00020	54
1979.470 \pm 0.027	0.00004 \pm 0.00020	-0.00035 \pm 0.00021	42
1980.439 \pm 0.018	-0.00003 \pm 0.00011	0.00011 \pm 0.00014	93
1981.464 \pm 0.021	-0.00020 \pm 0.00017	-0.00006 \pm 0.00016	64
1982.377 \pm 0.008	0.00025 \pm 0.00015	-0.00017 \pm 0.00018	50
1983.327 \pm 0.003	0.00028 \pm 0.00015	-0.00024 \pm 0.00020	82
1984.489 \pm 0.029	0.00006 \pm 0.00038	0.00003 \pm 0.00032	20
1985.278 \pm 0.000	0.00052 \pm 0.00040	-0.00142 \pm 0.00050	8
1986.536 \pm 0.006	0.00048 \pm 0.00020	-0.00021 \pm 0.00028	53
1988.597 \pm 0.010	-0.00088 \pm 0.00014	-0.00004 \pm 0.00022	37
1989.532 \pm 0.009	0.00020 \pm 0.00019	-0.00033 \pm 0.00022	48
1990.600 \pm 0.004	-0.00023 \pm 0.00011	0.00045 \pm 0.00016	102
1992.648 \pm 0.000	-0.00127 \pm 0.00046	0.00012 \pm 0.00057	8
1993.327 \pm 3.3x10 ⁻⁴	-0.00112 \pm 0.00025	0.00058 \pm 0.00040	17
1998.584 \pm 3.5x10 ⁻⁴	0.00040 \pm 0.00022	-0.00028 \pm 0.00027	58

TABLE 2.20
COMPARISONS OF ORBITAL PERIODS TO EFFECTIVE NYQUIST FREQUENCIES

Parameter	Frequency (yr ⁻¹)
12-year Orbital Period	0.083
20-year Orbital Period	0.050
Effective Nyquist Frequency, all observations	15.671
Effective Nyquist Frequency, nightly normal points	1.808
Effective Nyquist Frequency, annual normal points	0.325

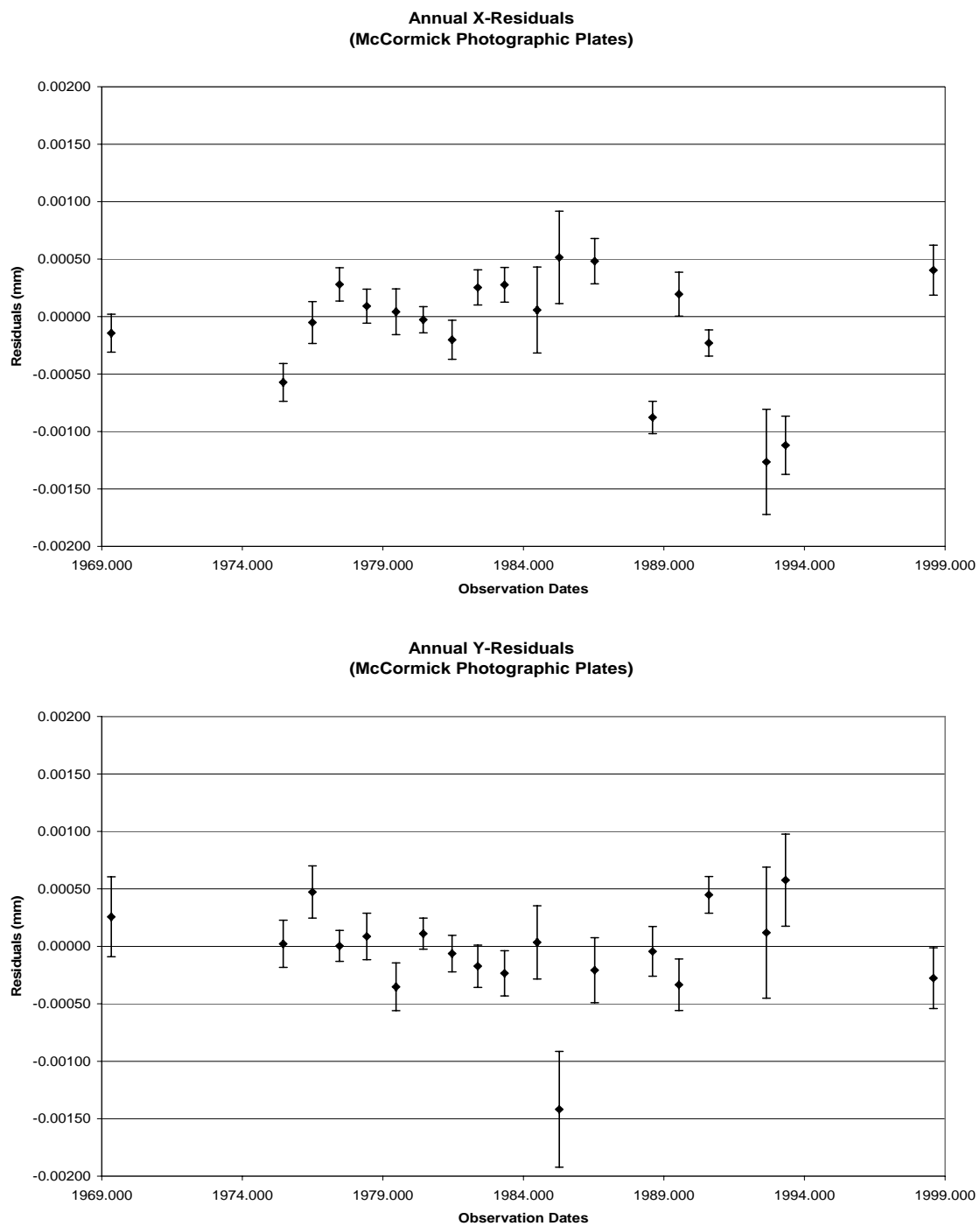


FIG. 2.9.—Annual X- and Y-Residuals for Barnard's Star. Top chart contains x-residuals averaged by year. Bottom chart contains y-residuals averaged by year.

The algorithm used searched frequencies up to four times the effective Nyquist frequency to produce plots of spectral power versus frequency. In addition, the algorithm calculated the false alarm probability, or the probability that noise rather than a true signal caused a particular peak in power (Press *et al.* 1996). Figure 2.10 shows that the power at no frequency is indicative of a signal at a significance level of 50% or better. The peak power of $3.951 \text{ millimeters}^{-2} (\text{mm}^{-2})$ in the x-residuals and of 2.966 mm^{-2} in the y-residuals occurs at a frequency of 0.051 yr^{-1} , corresponding to a period of 19 years. Van de Kamp (1982) suggested a range of 17–23 years for the outer planet. As intriguing as these peaks may be, the corresponding false alarm probabilities are 77% and 98%. In the periodogram for all x-residuals, this frequency corresponds to a local peak that has a greater than 75% chance of being real but the corresponding y-frequency is adjacent to a local peak that has a less than 50% chance of being real. In the periodogram for nightly x-residuals, the frequency is adjacent to a local peak that has a greater than 80% chance of being real but the corresponding y-frequency is a local peak that has a less than 50% chance of being real. The outer planet described by van de Kamp (1982) should produce a perturbation of Barnard’s Star on the order of 6 mas, which corresponds to about $0.3 \text{ } \mu\text{m}$ at the plate scale of the McCormick refractor. Therefore, these intriguing peaks are most likely caused by noise. Table 2.21 also lists the primary peaks in all the periodograms. Table 2.22 describes the features of all the periodograms that correspond to the 12 and 20-year orbits described by van de Kamp (1982).

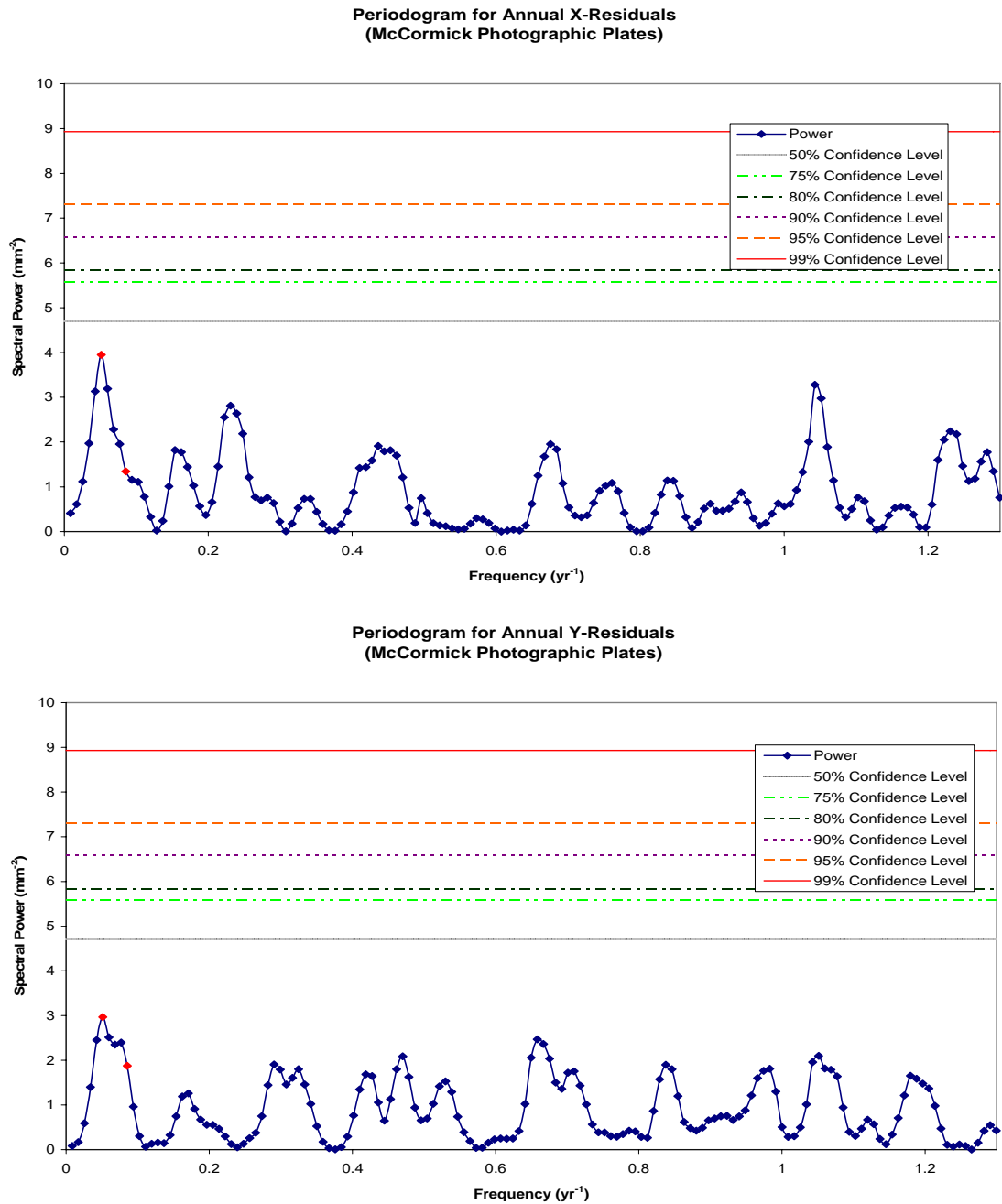


FIG. 2.10.—Periodigrams for Annual Barnard Star Residuals. Bottom left panel is a periodogram calculated for annual x-residuals. Bottom right panel is a periodogram calculated for annual y-residuals. Horizontal lines in the periodograms indicate likelihood that spectral peaks exceeding that power are real and not caused by noise only. Solid line is 99% confidence, dashed line 95% confidence, dotted line 90% confidence, dash dotted line is 80% confidence, dash double-dotted line is 75% confidence, and hashed line is 50% confidence.

TABLE 2.21
PRIMARY SPECTRAL PEAKS IN PERIDOGRAMS FOR BARNARD'S STAR

Periodogram	Freq. (yr ⁻¹)	Power (mm ⁻²)	Period (yr)	False Alarm Prob. (%)	Comments
All X	0.674	21.943	1.48	11	multiple of 6 month observing cycle?
All Y	1.978	18.474	0.5055	51	6 month observing cycle
Nightly X	0.674	13.285	1.48	72	multiple of 6 month observing cycle?
Nightly Y	1.978	7.961	0.5055	14	6 month observing cycle
Annual X	0.051	3.951	19	77	within 17–23 year range
Annual Y	0.051	2.966	19	98	within 17–23 year range

NOTE.—Van de Kamp (1982) suggests a range of 17–23 years for the orbit of the outer planet.

TABLE 2.22
PERIDOGRAM POINTS CORRESPONDING TO 12 AND 20-YEAR ORBITS

Periodogram	Freq. (yr ⁻¹)	Power (mm ⁻²)	Period ^a (yr)	False Alarm Prob. (%)	Comments
All X	0.051	9.616	20	< 25	local peak
All X	0.085	3.263	12	> 50	
All Y	0.051	4.386	20	> 50	adjacent to local peak
All Y	0.085	2.683	12	> 50	
Nightly X	0.051	7.766	20	< 20	adjacent to local peak
Nightly X	0.085	1.545	12	> 50	
Nightly Y	0.051	2.448	20	> 50	local peak
Nightly Y	0.085	0.968	12	> 50	
Annual X	0.051	3.951	19	> 50	primary peak within 17–23 yr range
Annual X	0.085	1.345	12	> 50	
Annual Y	0.051	2.966	19	> 50	primary peak within 17–23 yr range
Annual Y	0.085	1.873	12	> 50	

NOTE.—Van de Kamp (1982) suggests a range of 11.5–13 years for period of the inner planet and 17–23 years for the period of the outer planet.

as described by van de Kamp (1982) is less than any of effective Nyquist frequencies (f_{Neff})

$$f_{\text{Neff}} = \frac{N}{2T} \quad (2.4)$$

where N is the number of observations taken over total time (T). In comparison to the noisy periodograms calculated from the residuals produced by the MPRP, the periodograms calculated for a simulated perturbation clearly reveal a periodic signal. The 12-year period is easily recovered from the simulated, noise-free data as illustrated in Figure 2.4 and Figure 2.5.

2.3.4 Companion Mass Limits

The majority of the planet searches described above that failed to detect the planets described by van de Kamp (1982) were insensitive to such low mass planets in long period orbits. Assuming a circular orbit for a planet as described by van de Kamp, Kepler's third law describes the minimum detectable companion mass (M_p)

$$M_p = \left(\frac{M_*}{P} \right)^{\frac{2}{3}} \frac{\alpha}{\pi} \quad (2.5)$$

where M_* is the stellar mass ($M_* = 0.16 \pm 0.04 M_\odot$ per Delfosse *et al.* 2000), P is the period (12.0 ± 0.5 yr), π is the parallax (552 ± 7 mas), and α is the minimum detectable perturbation of the photocenter in milliseconds of arc.

Several α values have been suggested in literature, which predict sensitivity to planets with minimum masses between 1.1 and 2.1 M_{21} shown in Table 2.23. Campbell, Walker, & Yang (1988) settled on 10 mas based on astrometric studies of the stars

TABLE 2.23
MINIMUM DETECTABLE PERTURBATIONS AND CORRESPONDING MASSES

Alpha (mas)	Planetary Mass (M_{21})		Reference
10	1.1	± 0.2	1
20	2.1	± 0.3	2
21.3 ± 3.5	2.2	± 0.5	periodogram trial

REFERENCES.-(1) Campbell, Walker, & Yang 1988; (2) Heintz 1988

included in their radial-velocity program. Heintz (1988) suggested 20 mas because astrometric perturbations would reveal themselves as extremely large residuals.

To determine whether this investigation specifically could have detected the planets van de Kamp (1982) described, 1.1- μ m noise, assumed as typical of photographic data and represented by the MPRP residuals, was added to the positions shown in Figure 2.4 producing the positions shown in Figure 2.11. The McCormick residuals averaged $1.02 \pm 0.81 \mu$ m in x and $1.21 \pm 0.96 \mu$ m in y with corresponding maxima of 5.2 μ m and 5.5 μ m compared with the perturbation amplitudes of 0.72 μ m and 1.20 μ m, respectively described by van de Kamp (1982). The noise in the McCormick nightly normal points averaged $0.550 \pm 0.046 \mu$ m. As shown in Figure 2.12, the 12-year period is no longer obvious in the corresponding periodograms. Therefore, this study would not detect the planets van de Kamp described.

Using the same observation times and orbital parameters, the distance between the central star and its hypothetical planets was increased until the 12-year period began to appear in the related periodograms. Figure 2.13 shows a combination of displacements twice as large those associated with the inner planet

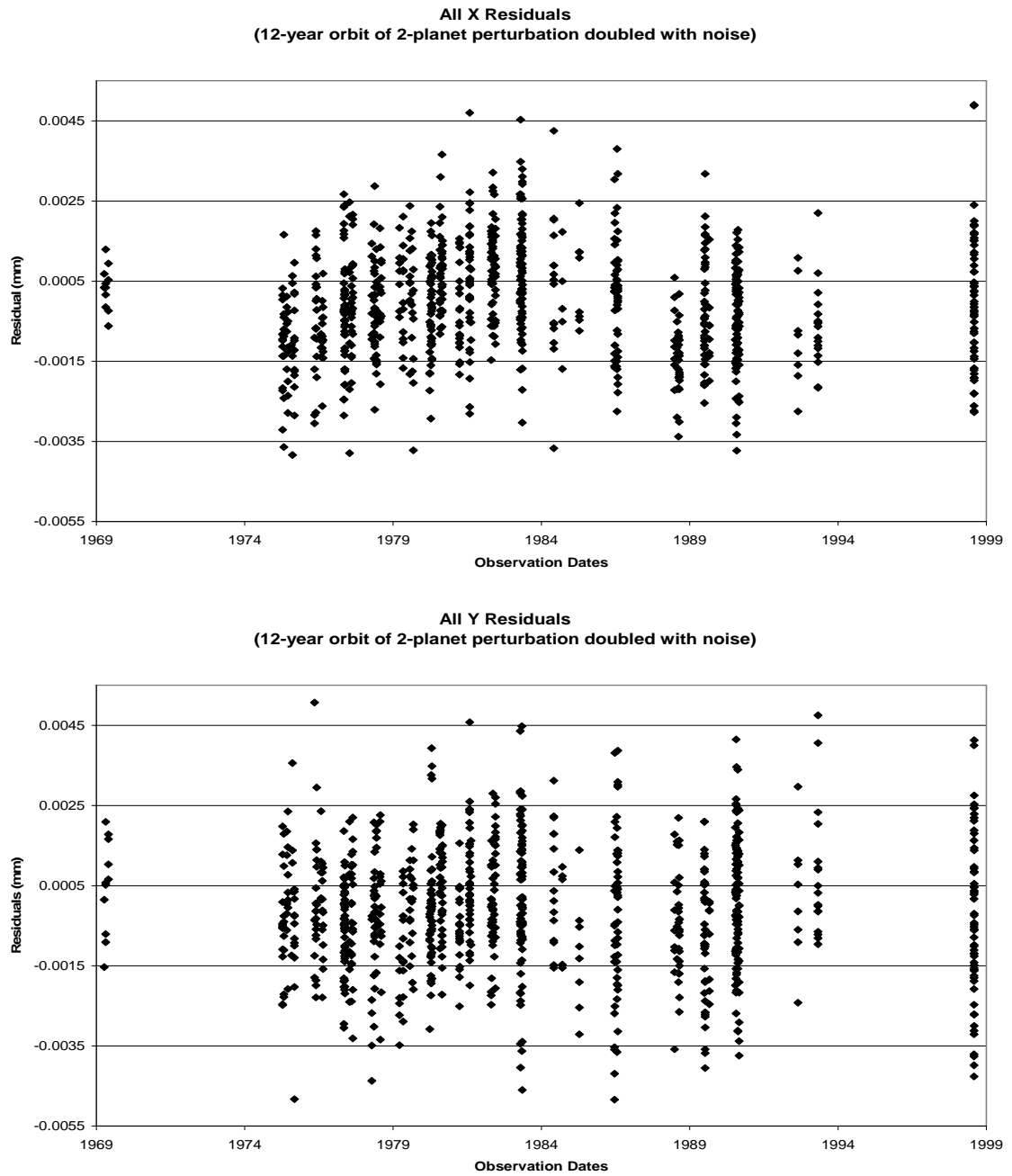


FIG. 2.13.— Theoretical Displacement Doubled with Noise Added. Top chart contains expected x-residuals combining the theoretical perturbation doubled and noise. Bottom chart contains expected y-residuals.

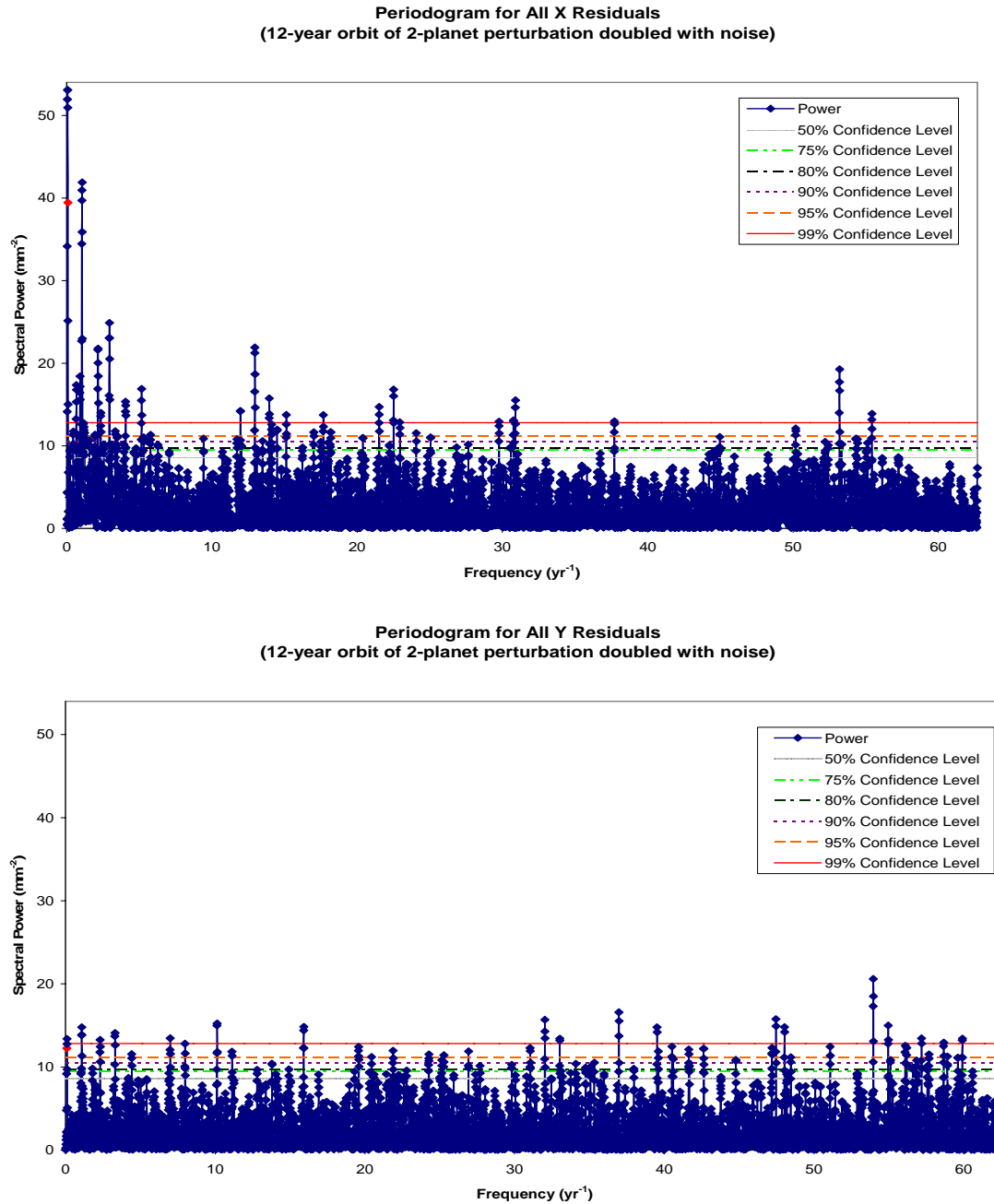


FIG. 2.14.—Periodograms of Theoretical Displacement Doubled with Noise Added. Top chart is a periodogram calculated for expected x-residuals. Bottom chart is a periodogram calculated for expected y-residuals. Horizontal lines in the periodograms indicate likelihood that spectral peaks exceeding that power are real and not caused by noise only. Solid line is 99% confidence, dashed line 95% confidence, dotted line 90% confidence, dash dotted line is 80% confidence, dash double-dotted line is 75% confidence, and hashed line is 50% confidence.

of the two-planet described by van de Kamp (1982) and the residuals of this study. The corresponding periodograms are displayed in Figure 2.14 as well.

For Figure 2.15, the orbital amplitude was increased three-fold. In the corresponding periodograms, also shown in Figure 2.16, the 12-year period begins to emerge from the noise represented by the McCormick residuals. In the periodograms for the all x- and nightly x-residuals, the maximum peak occurs at a frequency of 0.077 yr^{-1} , which corresponds to a period of 13 years. The maximum peak in the nightly y-residual periodogram occurs at a frequency of 0.085 yr^{-1} , which corresponds to a period of 12 years. For Figure 2.17, the planetary residuals were increased four-fold and the 12-year period is clearly visible in the periodograms of Figure 2.18. The maximum peak in each of the x-residual periodogram occurs at a frequency of 0.077 yr^{-1} , which corresponds to a period of 13 years. The maximum peaks in the all y- and nightly y-residual periodograms occur at 0.085 yr^{-1} , which corresponds to a period of 12. years. Continuing to increase the size of the perturbation does not change the frequency at which the x-residual periodograms peak but the power at 0.077 yr^{-1} increases.

From this simple test, we deduce that this study could detect a planet as small as $2.2 \pm 0.5 M_{24}$, which would produce a perturbation of $1.03 \pm 0.17 \mu\text{m}$, or $21.3 \pm 3.5 \text{ mas}$. A perturbation of this size is essentially the same as the $1.6\text{-}M_{24}$ planet initially reported by van de Kamp (1963b) and three times greater than the more massive planet

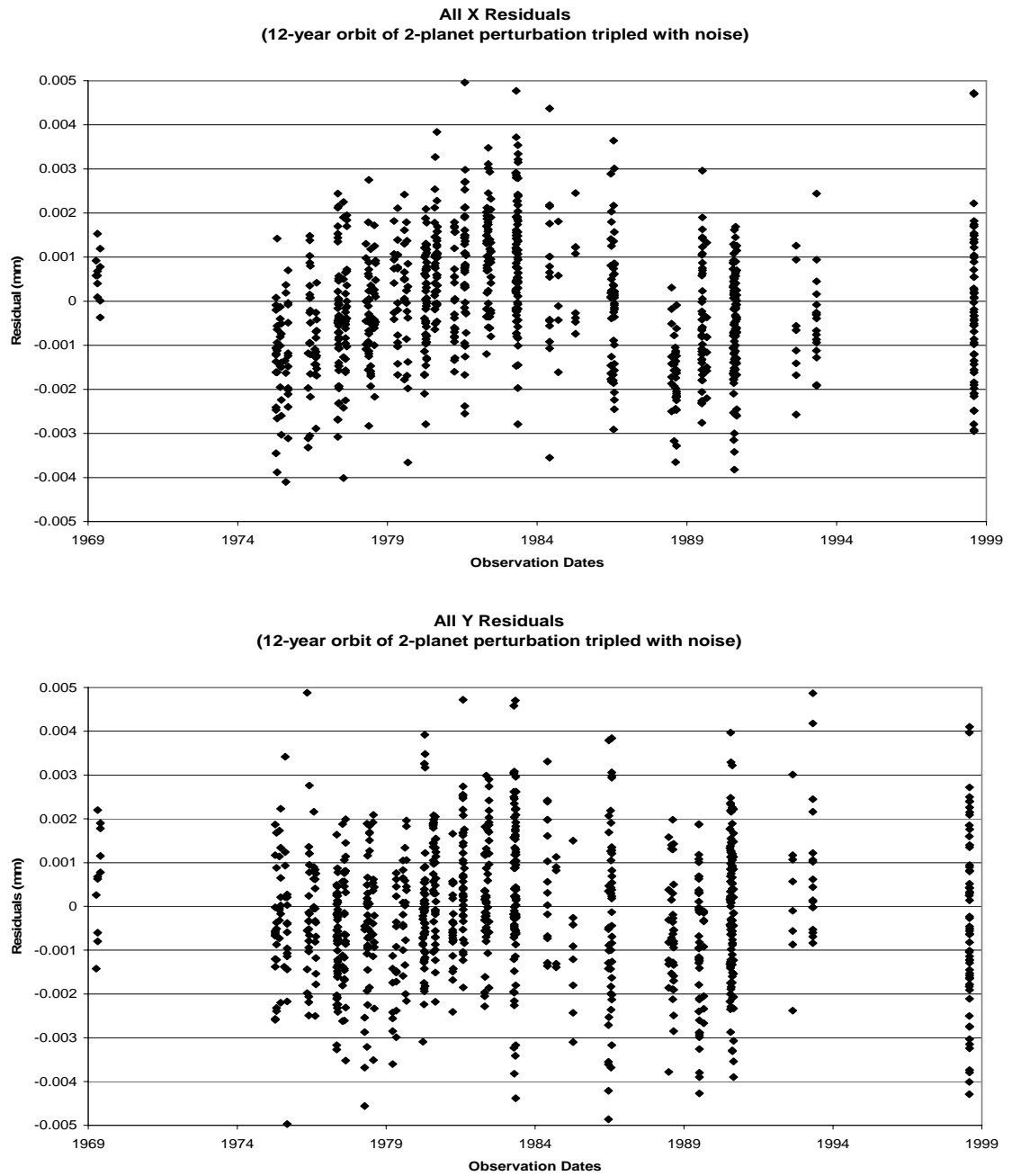


FIG. 2.15.— Theoretical Displacement Tripled with Noise Added. Top chart contains expected x-residuals combining the theoretical perturbation tripled and noise. Bottom chart contains expected y-residuals.

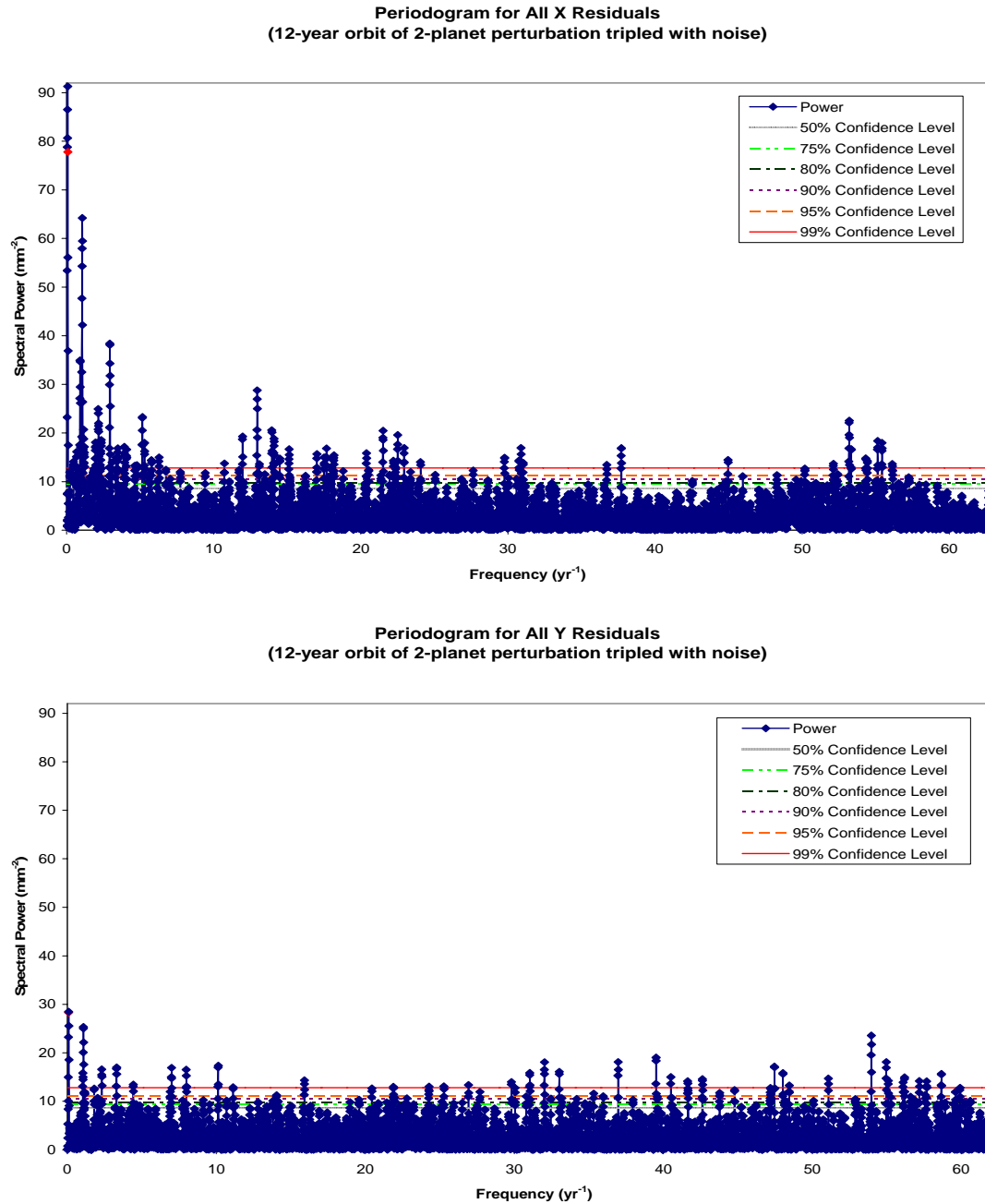


FIG. 2.16.— Periodograms of Theoretical Displacement Tripled with Noise Added. Top chart is a periodogram calculated for expected x-residuals. Bottom chart is a periodogram calculated for expected y-residuals. Horizontal lines in the periodograms indicate likelihood that spectral peaks exceeding that power are real and not caused by noise only. Solid line is 99% confidence, dashed line 95% confidence, dotted line 90% confidence, dash dotted line is 80% confidence, dash double-dotted line is 75% confidence, and hashed line is 50% confidence.

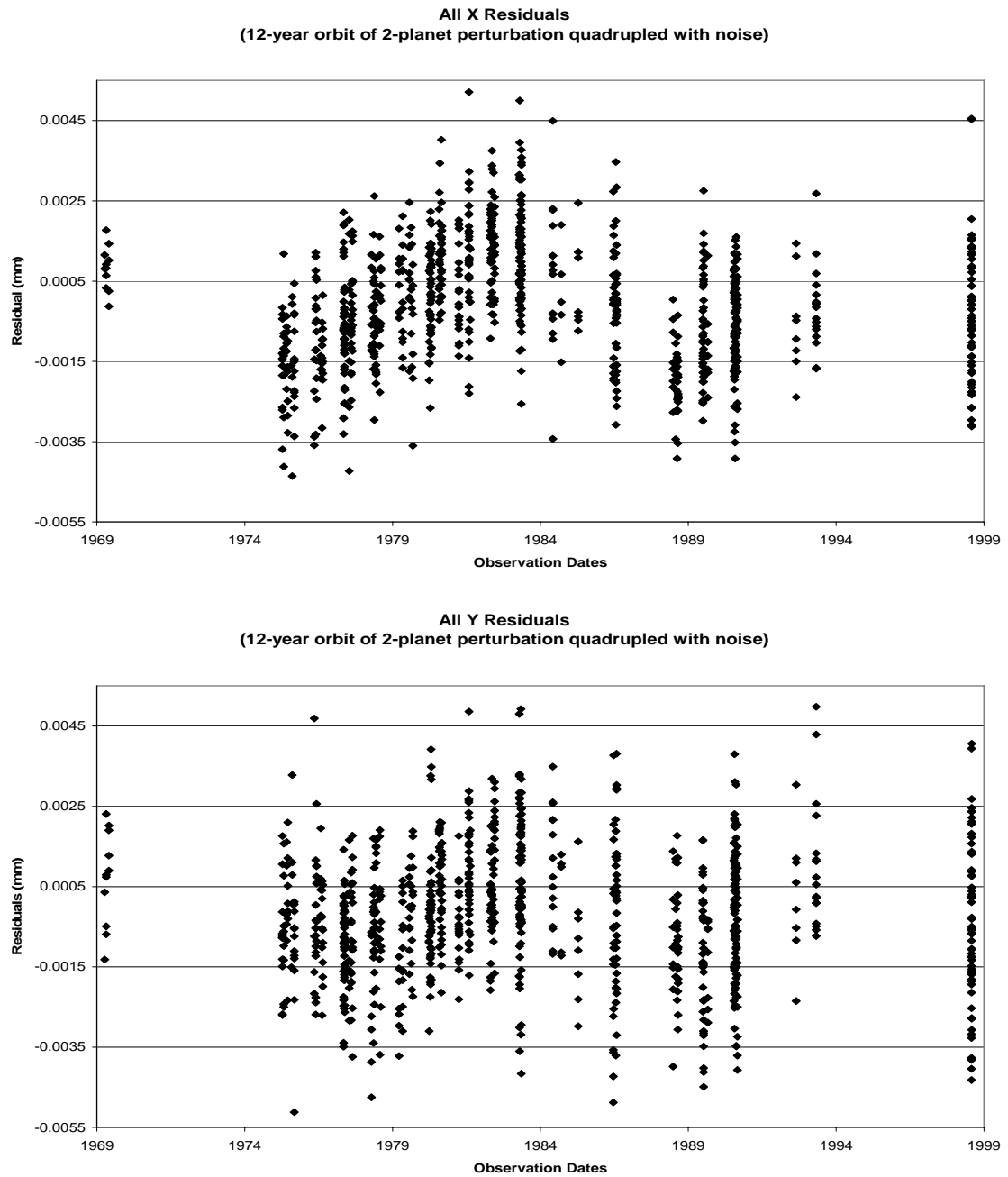


FIG. 2.17.— Theoretical Displacement Quadrupled with Noise Added. Top chart contains expected x-residuals combining the theoretical perturbation quadrupled and noise. Bottom chart contains expected y-residuals.

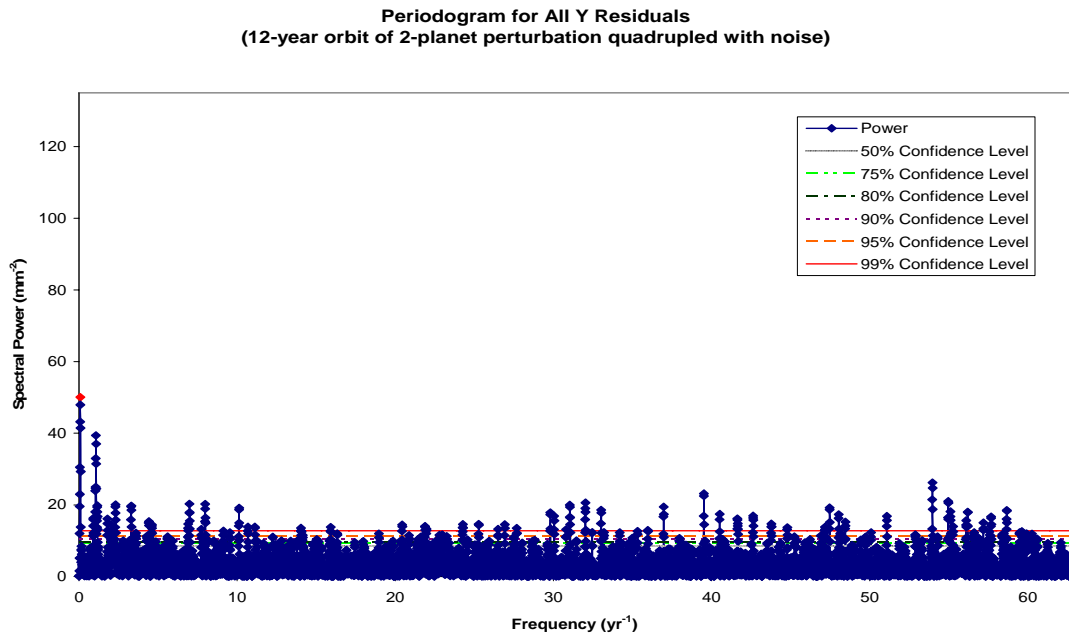
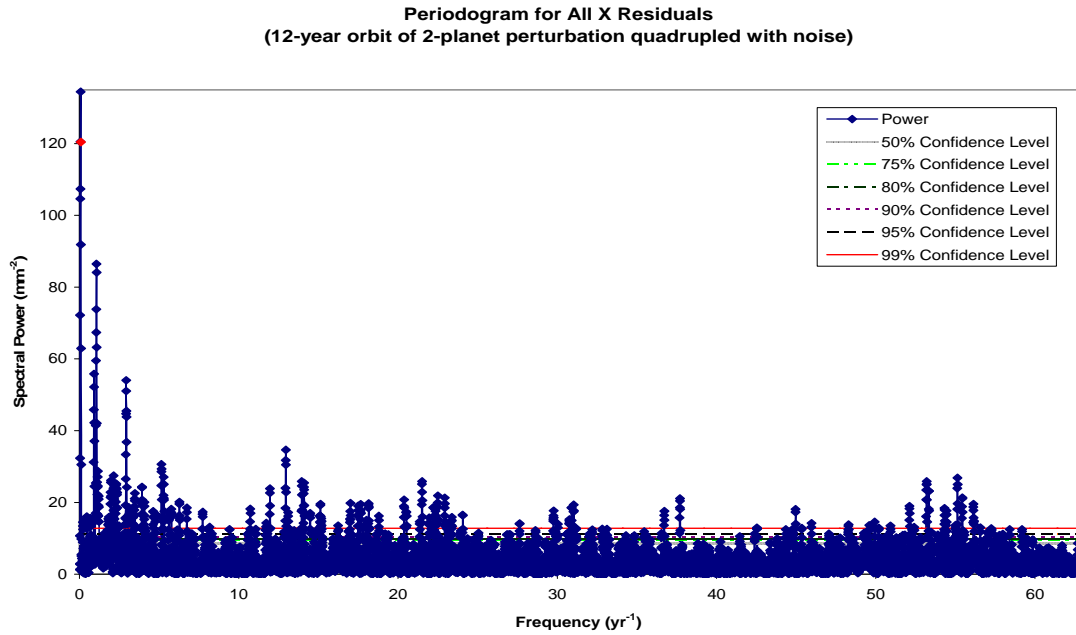


FIG. 2.18.— Periodograms of Theoretical Displacement Quadrupled with Noise Added. Top chart is a periodogram calculated for expected x-residuals. Bottom chart is a periodogram calculated for expected y-residuals. Horizontal lines in the periodograms indicate likelihood that spectral peaks exceeding that power are real and not caused by noise only. Solid line is 99% confidence, dashed line 95% confidence, dotted line 90% confidence, dash dotted line is 80% confidence, dash double-dotted line is 75% confidence, and hashed line is 50% confidence.

of the two described in his final analysis (1982). This low mass limit demonstrates the potential of astrometry to detect planets as well as larger companions.

2.4 DISCUSSION

This failure to replicate van de Kamp's results, while not unexpected considering the other searches to date, does not completely eliminate the possibility that such planetary companions exist. The Sproul Observatory material has similar precision to the McCormick photographic plates but the quantity of Sproul plates is unique. In 1982, van de Kamp analyzed nearly 20,000 exposures on 4,580 plates taken on 1,200 nights between 1938 and 1981. Although many other studies have attempted without success to confirm his results, no study has completely ruled out the possibility of low-mass companions to Barnard's Star.

Previous investigations of the photographic plate material at the McCormick have achieved similar negative results (Fredrick & Ianna 1985; Ianna 1995; Bartlett & Ianna 2001) despite van de Kamp's (1963a, 1969a) earlier analyses. Although containing far fewer observations than those taken at Sproul Observatory, McCormick is thought to have the second largest collection of Barnard's Star observations. In addition, the baseline of the McCormick observations represents slightly more than twice the period of the more massive, but shorter period, planet described by van de Kamp (1982). Furthermore, the McCormick refractor did not undergo significant physical changes, such as the modifications made to the Sproul refractor (van de Kamp 1975). Even with smoothing to annual normal points, the residuals to the proper motion and parallax solutions obtained in this study show the noise typical of photographic

plates. Nonetheless, this study further reduces the possibility that giant planets orbit Barnard's Star. Recent radial velocity results indicate such planets are even more unlikely leaving a remote possibility that smaller planets are present (C. McCarthy *et al.* 2007, in preparation).